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Multidimensional time and income poverty: well-being gap and minimum 2DGAP poverty intensity – German evidence

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Abstract This paper focuses on interdependent multidimensional poverty of time and income with its incidence and intensity. We introduce a Two Dimensional Minimum Poverty Gap (2DGAP) measure, which quantifies the shortest path to escape multidimensional poverty. The 2DGAP disentangles single poverty attribute gaps while assuring their interdependence; an important issue for targeted antipoverty policies. Besides income, we include *genuine personal leisure time* with social participation reflecting Sen's capability approach. The interdependence of multidimensional poverty is estimated by a CES-type well-being function with individual German data. The empirical results of Germany's "working poor" emphasize the importance of time with social participation aspects in the multidimensional poverty discussion.

Keywords Interdependent multidimensional time and income poverty · Genuine personal leisure time · Union and compensation approach · Minimum multidimensional poverty gap (2DGAP) · Extended economic well-being · Satisfaction/happiness · Working poor · CES well-being functon · German Socio-Economic Panel · German Time Use Survey 2001/02

1 Introduction

There is growing interest in extending the traditional unidimensional income poverty concept with a multidimensional poverty approach (see e.g. [21, 29, 50]). The measurement of

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multidimensional poverty requires two main decisions: which poverty dimensions should be part of the multidimensional poverty index, and how should these dimensions be aggregated to a poverty index? As to poverty dimensions, for example, the EU multidimensional indicators of social inclusion, known as 'Laeken indicators', include educational disadvantages, health inequalities, unemployment and worklessness [4].[1] As to the aggregation across single poverty dimensions, the so-called counting approach [1, 4, 10] is based on the number of dimensions on which a person is considered deprived. On the other hand, all single poverty dimensions can interact, and interdependence accounts for compensation among single poverty dimensions within the multidimensional approach. Yet, the problem with any aggregation across dimensions into a multidimensional poverty index is that each single poverty dimension is no more transparent. Such information for single poverty attributes, however, is particularly important for policy makers developing targeted antipoverty policies.

Our study contributes a new approach to measuring multidimensional poverty by individual poverty attributes that are transparent. The interdependence of single poverty dimensions and its degree of compensation is modelled by a constant elasticity of substitution (CES) type well-being function. The contribution of each single poverty attribute is made transparent by a novel multidimensional poverty intensity index, the minimum multidimensional poverty gap, called the minimum "2DGAP", which is based on the minimum Euclidian distance between an individual's allocation and a pre-defined poverty threshold. The minimum 2DGAP (MDGAP with m poverty dimensions) maps the aggregated well-being gap onto the single poverty dimensions, and quantifies a unique solution as the shortest path to escape multidimensional poverty.

Specifically, this index separates the intensity of each single poverty dimension while ensuring the interdependence of multidimensional poverty attributes. As a unique solution the minimum 2DGAP overcomes the ambiguity of the single attributes in an aggregated well-being gap with a single value; knowing the intensities of single poverty attributes in the minimum 2DGAP allows policy makers targeted antipoverty policies.

The evaluation by a population of the interdependence of the multidimensional poverty attributes – as will be in our case - result in an explicit multidimensional 2DGAP poverty intensity index and its attributes (say time and income) in particular. The now transparent minimal combination of poverty attributes to lift a person (or a group of persons) out of poverty tells a policy maker how the population evaluates the relative importance of and interdependence among different poverty attributes. Knowledge of their relative importance might then influence the mixture of antipoverty policies. Yet, this kind of evaluation is different to actual costs of escaping poverty. As long there is no additional knowledge about such costs/prices, nothing can be said about any economic efficiency of those policies. However, without taking into account a population's evaluation of the poverty attributes' interdependence with its 2DGAP components, an important policy anchor would be missing.

In our application and with respect to single poverty dimensions, we argue that time, in addition to income, is an important well-being and poverty dimension. This is because time is a basic resource that is necessary for any activity. In particular, we propose to add *genuine personal leisure time* as a prominent poverty dimension, because it is a condition for social

[1] Another example for a multidimensional measure of well-being is the UN Human Development Index (HDI) which measures poverty, literacy, education, life expectancy, and other factors aggregated in a single index (http://hdr.undp.org/en/statistics/hdi).

participation and social inclusion, a point highlighted by Amartya Sen's [42] capability approach.

The empirical application is based on data from the German Socio-Economic Panel (SOEP) which provided individual satisfaction data for the estimation of the CES well-being function, and the German Time Use Study (GTUS) 2001/02 with its detailed time use diary data. Though our CES well-being function for measuring well-being is only slightly different compared to those of Bourguignon and Chakravarty [11] and Lugo and Maasoumi [31, 32], our novel empirical evaluation of compensation/substitution does not arbitrarily assume a specific compensation when quantifying the CES parameters. Our CES parameters are based on the total German population, and are econometrically estimated by individual satisfaction data.

Our findings add to understanding the poverty situation of the "working poor", who are a socially important and growing group, at least in Germany [2, 40].

The study has a theoretical and an empirical part and is structured as follows: The *theoretical part* starts by measuring multidimensional poverty in Section 2, with multidimensional poverty axioms (Section 2.1), identification under the strong (union approach) and weak focus axiom (compensation approach) in Section 2.2, and develops a multidimensional Foster-Greer-Thorbecke (FGT) type well-being measure of poverty risk and poverty gap. The new multidimensional minimum poverty 2DGAP is introduced in Section 3. Section 3.1 presents the concept and illustration (Section 3.1), definition and calculation (Section 3.2) and single poverty attribute intensities, aggregation and extensions (Section 3.3).

The *empirical part* is divided in two sections: Section 4 justifies and discusses the applicability of the concepts of time and income poverty and their interdependence with respect to the CES well-being multidimensional poverty approach and threshold. Section 5 presents the empirical results with the data bases (Section 5.1), the empirical poverty lines (Section 5.2), uni- and multidimensional well-being based poverty results (Section 5.3) and minimum multidimensional 2DGAP poverty findings (Section 5.4) for the active German population. The last section provides a further discussion, a conclusion and summarizes the main contribution of the study.

2 Measuring multidimensional poverty

This theoretical section is about measuring multidimensional poverty. Based on respective axioms multidimensional identification and intensity well-being measures are discussed followed by introducing the multidimensional minimum poverty 2DGAP.

In the unidimensional context, poverty indices based on certain desirable axioms have been extensively discussed (see e.g. [18, 53]). In the multidimensional context, axiomatically based poverty indices are a more recent development (see [11, 19, 47] or the survey by [17], chapters 5, 6). However, discussion about this and a more empirical application is still in its infancy.

2.1 Multidimensional poverty axioms

Our well-being based multidimensional poverty measure in Section 2.3 is based on poverty axioms. In line with the approach by Maasoumi and Lugo [33] the majority of unidimensional poverty axioms are also transferable to our multidimensional approach: The poverty axioms of symmetry, monotonicity, continuity, principle of population, scale invariance and subgroup decomposability ([11], p. 29, [33], p. 5). The focus axiom in the multidimensional context, however, is of particular importance for the development of our arguments and will be discussed apart.

The focus axiom in the unidimensional context claims that a poverty index has to be independent of non-poor persons' poverty attribute quantities. However, with a multidimensional context two different approaches for the focus axiom are conceivable. On the one hand, the multidimensional focus axiom could demand that the multidimensional poverty index be independent of quantities lying above the single dimension thresholds. On the other hand, it could only be required that the index be independent of non-multidimensional poor persons' attribute quantities.

The former requirement is called the *strong focus axiom*, while the latter is named *weak focus axiom* [11]. The consideration of these two axioms corresponds to the question of whether deprivation in one dimension could be compensated for by another dimension's quantities above the dimension threshold. The weak focus axiom allows such a substitution for all poverty ranges while the strong focus axiom does not for all poverty ranges.

Moreover, in a multidimensional case and for our multidimensional poverty measure, it is debatable whether a so-called correlation increasing switch should raise or reduce the multidimensional poverty index. A correlation increasing switch is a change of quantities between two multidimensional poor persons in a deprived dimension that increases the correlation between dimensions. After such a switch, strong deprivation in one dimension is increasingly attended by strong deprivation in the other. Depending on the empirical relationship between poverty dimensions, one could expect an increase or a decrease of the multidimensional poverty index. The respective correlation situation within our analysis is described in Section 2.3.

2.2 Multidimensional poverty identification

To identify multidimensional poor individuals usually a definition of two kinds of thresholds are required. On the one hand, a poverty threshold for each poverty dimension is needed to count e.g. the number of deprived dimensions for each individual. On the other hand, one has to determine in how many dimensions an individual has to be deprived in order to be judged multidimensionally poor. Two extreme approaches are distinguished. Following the so called *union approach (strong focus axiom)*, a person is judged multidimensionally poor as soon as he or she is deprived in at least one dimension (see Fig. 1 for the two dimension case). The *intersection approach*, in contrast, judges an individual to be multidimensionally poor when she or he is deprived in all dimensions. Intermediate concepts are conceivable as well (see e.g., [1, 4]).

Ultimately the selection of the union, intersection or intermediate approach as the identification strategy depends on the relationship between poverty dimensions. This raises the fundamental question whether a substitution/compensation between poverty dimensions is possible. Given a substitutive situation, the intersection approach seems to be preferable. In an intersection approach the deficit in one dimension is compensated for by another. If a complementary interaction between poverty dimensions is given where deprivation in one dimension cannot be compensated by the other attribute, then the union approach would be appropriate (see e.g., [4]).

Against this background the issue arises whether at least a limited substitution should be considered in the identification of a poor person. In the vast majority of cases, poverty dimensions are neither perfect substitutes nor perfect complements but something between these two extremes. Accordingly, the deficit in one dimension can be compensated to

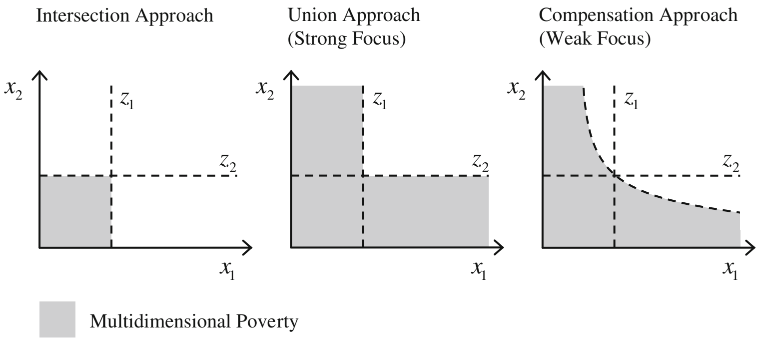

Fig. 1 Identification of multidimensional poverty with the intersection, union and compensation approach
Note: x_1 and x_2 are the quantities of the first and second dimension while z_1 and z_2 are the corresponding poverty dimension thresholds
Source: Own figure

a limited extent and with diminishing returns by another attribute. Union and intersection approaches as identification strategies seem to be too rigid for most cases ([13], p. 2; [32], p. 25).

The empirical question is whether and to which extent a poverty gap in one dimension might be compensated by higher quantities in the other dimension. If a gap in one dimension can be compensated by another dimension's quantity above the dimension threshold, then a person is off poverty (right picture in Fig. 1, not shaded area). If such a gap cannot be compensated by another dimension's quantity then the person will be called multidimensional poor. We will call such an approach a *compensation approach (weak focus axiom)* because substitution/compensation is allowed for all ranges in one dimension given poverty in the other dimension.

Because substitution/compensation is allowed in the compensation approach (weak focus property) as well as in the union approach (but limited to the intersection approach) we will call this poverty situation *interdependent multidimensional poverty* (IMD poverty).

Considering then the multidimensional poverty line (based on two dimensions as in Fig. 1), there is general agreement that an individual who is deprived in both dimensions should be judged as multidimensionally poor, while an individual who is not deprived in any dimension should not be judged as multidimensionally poor. Accordingly, a multidimensional poverty line that accounts for at least limited substitution and diminishing returns has to run through the intersection of the dimension thresholds (z_1, z_2) as in Fig. 1.

2.3 Multidimensional poverty intensity: headcount ratio and multidimensional poverty well-being gaps

After identifying multidimensionally poor individuals in the previous section, the question is how to capture the extent of poverty, i.e., the intensity of poverty, within a multidimensional poverty index. In the unidimensional context the Sen-Shorrocks-Thon (SST index)[2]

[2]See Osberg and Xu [39] based on Sen [41], Shorrocks [45], Thon [46].

or the Foster-Greer-Thorbecke [23] (FGT) indices are well-known. In the case of the unidimensional FGT index, the individual poverty function

$$q_i = \max\left[\frac{z - Y_i}{z}; 0\right] \tag{1}$$

measures the poverty gap as a relative deviation of the well-being indicator $Y_i \geq 0$ (income, say) to the defined poverty threshold $z \geq 0$ (income, say). The aggregation over all individuals ($i = 1, \ldots, n$) yields the unidimensional FGT poverty index

$$P(Y, z) = \frac{1}{n}\sum_{i=1}^{n}(q_i)^{\alpha} = \frac{1}{n}\sum_{i=1}^{n}\left[max\left(\frac{z - Y_i}{z}; 0\right)\right]^{\alpha}, \tag{2}$$

and α indicates the poverty risk aversion: the higher the parameter, the more sensitive the index is to strong deprivations. For $\alpha = 0$, the headcount ratio results, $\alpha = 1$ corresponds to the poverty gap and $\alpha = 2$ represents a quadratic poverty gap.

In the multidimensional context, particularly Lugo and Maasoumi [32] and Bourguignon and Chakravarty [11] embrace all dimensions in their multidimensional poverty indices. Lugo and Maasoumi [32] attempt to transfer the unidimensional FGT index to the multidimensional framework. They classify two aggregation approaches: one by "shortfall of well-being" (aggregate poverty line approach) and one by "well-being of the shortfalls"[3] (component poverty line approach). Both of them might be analyzed under the strong or weak focus poverty axiom. The first one relies on individual well-being compared to well-being at the threshold intersection, where well-being is measured as the output of a production type well-being function with two (or more) input factors allowing substitution. In the second one, the relative differences between the individual dimensional attributes and their thresholds are the respective input factors of the well-being function.

Based on the data in our empirical application, we evaluate the individual's income and time situation by developing levels. Accordingly we concentrate on the "shortfall of well-being" approach with regard to levels, rather than relative deviations in the well-being function.

The interdependence of the single poverty attributes within the individual well-being indicator measured by a Constant Elasticity of Substitution (CES) type function is already used by several authors[4] (e.g., [32], pp. 12, 16, [16], [11], p. 38, or [12], p. 23) and is captured there by a CES function V_i^* with

$$V_i^* = \left[w_1(x_{1i})^{\beta} + w_2(x_{2i})^{\beta}\right]^{\frac{1}{\beta}},$$

where β describes the level of substitutability with $\beta = 1$ for perfect substitution and $\beta = \infty$ for non-substitutes. Similar and in line to them but with a slightly more flexible CES-type well-being function our individual well-being indicator V_i evaluates the interdependencies of both poverty dimensions by:

$$V_i = \gamma\left[w_1(x_{1i})^{-\rho} + w_2(x_{2i})^{-\rho}\right]^{\frac{\upsilon}{-\rho}} \quad weak\ focus\ axiom\,, \tag{3}$$

and

$$V_i = \gamma\left[w_1(\min[x_{1i}, z_1])^{-\rho} + w_2(\min[x_{2i}, z_2])^{-\rho}\right]^{\frac{\upsilon}{-\rho}} \quad strong\ focus\ axiom\,, \tag{4}$$

[3]Which corresponds to the Bourguignon and Chakravarty [11] multidimensional poverty index.

[4]Originated from production theory, see Arrow et al. [3].

with the substitution elasticity $\sigma = 1/(1+\rho)$, ρ as a curvature parameter of the isopoverty contours with $\rho \neq 0$, γ as a constant, υ as returns to scale, x_{1i} and x_{2i} as the input (poverty attribute) quantities and z_1 and z_2 as the thresholds of the first and second poverty dimension, and the input coefficients w_1 and $w_2 = 1 - w_1$ as distribution and weighting parameters describing the skewness of the isopoverty contours. With regard to a meaningful interpretation the coefficient w_1 (and thus w_2, too) is defined for the $0 < w_1 < 1$ interval.[5]

Why do we use such a CES-type well-being function? Within the CES well-being function, substitution/compensation is possible between all levels of the poverty dimensions. Constant elasticity in general assumes that any poverty attribute pair, like time and income as in our case, is held together by its degree of substitution, regardless of the level of well-being. However, substitution is different between different proportions of income and time. This is indeed a quite flexible formulation.[6]

Beyond the substantial meaning of the constant γ as some basic log (well-being), and with the returns to scales υ as showing the effects from a proportional change in all inputs (where all inputs increase by a constant factor), concerning the later econometric estimation our CES well-being function provides a better goodness of fit within the empirical estimation (see Section 5.2).

Note that the arguments in the strong focus case with min(.) have restricted the input levels to the poverty lines. This is not the case under the weak focus axiom. Accordingly, under strong focus axiom a substitution between input factors is not possible above the dimension thresholds (see Fig. 1). However, and as mentioned above, under the weak focus axiom, substitution is possible in all regimes below the multidimensional isopoverty threshold.

The *multidimensional poverty line*

$$V_z = \gamma \left[w_1 (z_1)^{-\rho} + w_2 (z_2)^{-\rho} \right]^{\frac{\upsilon}{-\rho}} \quad weak\ and\ strong\ focus\ axiom \tag{5}$$

is the aggregate poverty line under the weak and strong poverty axiom and is the isopoverty contour crossing the threshold intersection at $z = (z_1, z_2)$ (see Fig. 1).

The *multidimensional poverty function* is similar to the unidimensional FGT poverty measure, but with well-being units rather than income units as arguments in the relative gaps, and is represented as:

$$q_i = \max \left[\frac{V_z - V_i}{V_z}; 0 \right]. \tag{6}$$

The aggregated (across individuals) multidimensional FGT poverty measure corresponding to the Lugo and Maasoumi [32] procedure then is

$$P(V,z) = \frac{1}{n} \sum_{i=1}^{n} (q_i)^{\alpha} = \frac{1}{n} \sum_{i=1}^{n} \left[max \left(\frac{V(z_1,z_2) - V(x_{1i}, x_{2i})}{V(z_1,z_2)}, 0 \right) \right]^{\alpha} \quad weak\ focus\ axiom \tag{7}$$

$$P(V,z) = \frac{1}{n} \sum_{i=1}^{n} (q_i)^{\alpha} = \frac{1}{n} \sum_{i=1}^{n} \left[max \left(\frac{V(z_1,z_2) - V[\min(x_{1i}, z_1), \min(x_{2i}, z_2)]}{V(z_1,z_2)}, 0 \right) \right]^{\alpha} \quad strong\ focus\ axiom \tag{8}$$

with $\alpha = 0$ delivering the multidimensional headcount, $\alpha = 1$ an average relative poverty gap in well-being units applied to the total population which measures poverty intensity, and $\alpha > 1$ respecting a higher aversion against strong deprivations.

[5] For w_1 at the interval limits would cancel one of the attributes; for $w_1 < 1$ or $w_1 > 1$ one poverty attribute would always diminish well-being, which is neither desirable.

[6] See further discussion, interpretation and justification in Merz and Rathjen [35].

Since we use only a slightly different CES well-being function from that of Lugo and Maasoumi [32], the multidimensional FGT poverty measures for strong and weak focus fulfil the above mentioned axioms under the constraint, of course, that the compensation approach (weak focus) does not satisfy the strong focus axiom [32]. In addition, with the above aggregation over the poverty dimensions and the poverty line V_z we utilize the information theory rationale behind the respective entropy divergence minimization ([32], p. 10).

Concerning the aggregation of the individual multidimensional poverty index Bourguignon and Chakravarty [11], p. 42 presumed that the weak focus axiom would rule out functional forms of poverty indices that are additive as well as the CES-like P_α^θ measures. Their P_α^θ measure is a Foster-Greer-Thorbecke type of multidimensional poverty measure based on dimensional shortfalls. However, the FGT type multidimensional poverty measure we use with Eq. 7 relies on well-being shortfalls. In this and our case the "Aggregate Poverty Line (APL)" might be defined as used in Eq. 7 being consistent with the weak focus axiom ([32], p. 11).

Concerning the *non-decreasing/non-increasing poverty under correlation increasing switch*: The multidimensional FGT with our CES well-being function specification further fulfils *non-decreasing poverty under correlation increasing switch* ([11], p. 31) if $\alpha \geq -\rho$ (assuming $\upsilon = 1$), respectively, fulfills *non-increasing poverty under correlation increasing switch* if $\alpha \leq -\rho$ (assuming $\upsilon = 1$).

3 Minimum multidimensional poverty gap: minimum 2DGAP - a measure in the single dimensional space

The virtue of measuring multidimensional poverty by well-being as above is that it respects and quantifies the interdependence of multiple well-being attributes by a well-being index with a single value. However, such an aggregation over dimensions into a single value well-being index is criticized and questioned if it is still measuring "multidimensional" poverty. Transparency for the single attributes within the multidimensional approach is demanded which would allow a targeted attribute specific antipoverty policy.

The main motivation for the following multidimensional gap development and proposal thus is to unfold the single attributes of a well-being gap to obtain a unique but multidimensional intensity measure with transparent single attributes.

3.1 Minimum multidimensional poverty gap (minimum 2DGAP) – concept and illustration

As discussed, in the compensation (weak focus) approach all dimensions are combined and weighted via the respective CES well-being function delivering a one value well-being level and index. Figure 2 (top) describes the well-being function with $V_z = V(z_1, z_2)$ as the well-being level at the threshold isopoverty line of the single poverty thresholds $z = (z_1, z_2)$ and $V_i = V(x_{1i}, x_{2i})$ as an individual well-being level with the individual poverty attributes $x = (x_1, x_2)$ (two dimensional case). The difference $V_z - V_i$ is the multidimensional poverty well-being gap under discussion.

The mapping of the multidimensional well-being to its (two) single dimensional space allows another appealing integrated approach for describing multidimensional poverty intensity. It consists of a unique distance between the individual situation and the poverty

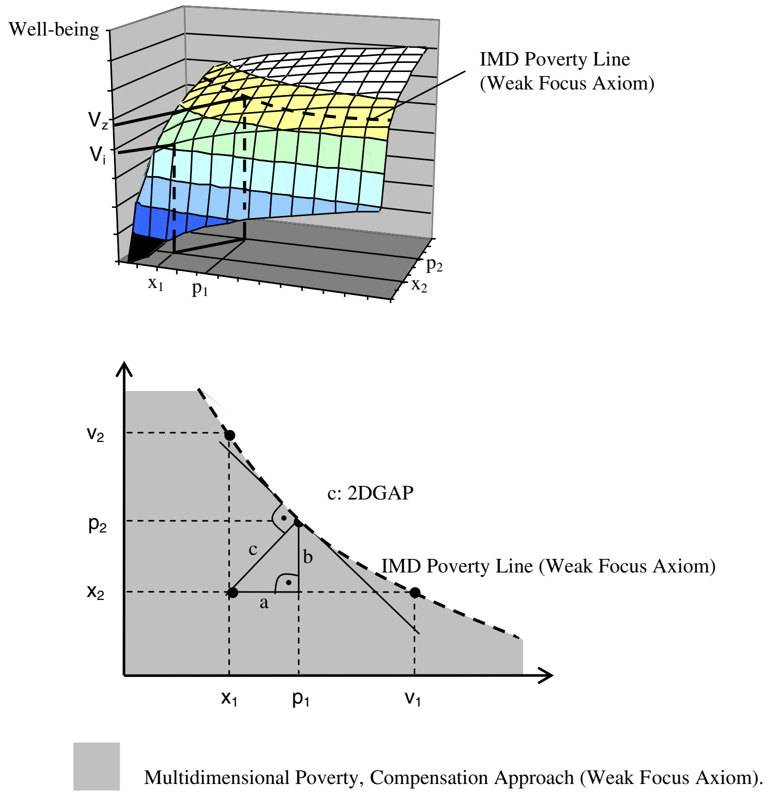

Fig. 2 Multidimensional poverty well-being gap and minimum 2DGAP as a mapping onto its dimensions Source: Own figure

threshold which at the same time provides the contribution of the single poverty attributes to the interdependent multidimensional poverty index.

As an illustration consider the two-dimensional case from the compensation approach and its attributes' space as in Fig. 2 and regard the poverty situation at $x = (x_1, x_2)$ for an individual. With respect to both dimensions there is a fan of distances from that point $x = (x_1, x_2)$ to the IMD weak focus isopoverty threshold. Each distance yields the same well-being difference $V_z - V_i$ (third dimension). However, each distance defines different single attribute input intensities in order to escape multidimensional poverty.

The *shortest* path between $x = (x_1, x_2)$ and the corresponding point $p = (p_1, p_2)$ at the isopoverty threshold contour is prominent.[7] It requires the minimum input intensities in a sense of a minimum combined input "length" in order to escape multidimensional poverty. A natural measure for that length is the Euclidian distance of the single attributes $c = [a^2 + b^2]^{0.5}$, with the distances a and b as the single poverty attribute gap intensities (see Fig. 2).

[7] Lugo and Maasoumi ([31], p. 14, [32], p. 12) already mention a distance from an individual point to the isopoverty line as the 'closest point' at the isopoverty line in the multidimensional case, however they do not determine any further characteristics and properties of that distance.

The solution for the shortest (or closest) length then is characterized by the orthogonal path from the tangent at $p = (p_1, p_2)$ to $x = (x_1, x_2)$. We call the distance c the minimum multidimensional poverty 2DGAP (for two poverty dimensions).

3.2 Minimum multidimensional poverty gap (minimum 2DGAP) – definition and calculation

For any individual point $x = (x_1, x_2)$ in the two dimensional poverty space under the weak focus CES-type isopoverty threshold the *minimum multidimensional poverty 2DGAP c* is defined as the shortest length (Euclidean norm) to the isopoverty line.

The shortest length is the linear path orthogonal to the slope at the respective point $p = (p_1, p_2)$ on the CES-type isopoverty threshold:

$$c = \|c\| = \left[(p_1 - x_1)^2 + (p_2 - x_2)^2\right]^{0.5} = \left[(p_1 - x_1)^2 + (f(p_1|V_2) - x_2)^2\right]^{0.5} = \text{min!} \tag{9}$$

where $p_2 = f(p_1|V_z)$ is the isopoverty contour ordinate with regard to ordinate values x_2 (say time) of the CES multidimensional well-being function $V_z = \gamma\left[w_1(z_1)^{-\rho} + w_2(z_2)^{-\rho}\right]^{\frac{\upsilon}{-\rho}}$ as

$$f(p_1|V_z) = \left(\left(\left(\frac{V_z}{\gamma}\right)^{-\frac{\rho}{\upsilon}} - w_1 p_1^{-\rho}\right)/w_2\right)^{-\frac{1}{\rho}}. \tag{10}$$

The solution p_1 of the minimizing problem at point $(p_1, p_2 = f(p_1|V_z))$ of the isopoverty contour – where the slope $c'_\perp$ (marginal rate of substitution) is orthogonal to the slope of c – then allows the calculation of c by Eq. 9 for $x = (x_1, x_2)$.

The orthogonal property allows another solution route via the respective slopes

$$c'_\perp(p_1) - f'(p_1|V_z) = 0, \tag{11}$$

by solving

$$-\frac{p_1 - x_1}{f(p_1|V_z) - x_2} + \left(\left(\left(\frac{V_z}{\gamma}\right)^{-\frac{\rho}{\upsilon}} - w_1 p_1^{-\rho}\right)/w_2\right)^{-\frac{1}{\rho}-1}\left(\frac{w_1}{w_2}\right)p_1^{-\rho-1} = 0 \tag{12}$$

with $c'_\perp = -1/c' = -a/b$, $c' = \frac{\partial c}{dp_1} = \tan(\alpha) = b/a$ and $\tan(\alpha + 90°) = -1/\tan(\alpha) = -a/b$.

The solution p_1 of the nonlinear Eq. 12 then allows to calculate c again by Eq. 9 for a respective $x = (x_1, x_2)$. The solution of Eq. 12 might also be found by an explicit iterative procedure in the interval $[x_1, \upsilon_1]$[8] of changing c slopes until the slope of the isopoverty line is orthogonal to the slope of path c which crosses (x_1, x_2).[9]

Since the proposed CES well-being function is well behaved, there is always a unique solution for the minimum 2DGAP (distance c).

[8] υ_1 is the abscissa value (say income) of the isopoverty contour of V_z equal x_2:

$$\upsilon_1\left(g(\upsilon_1|V_z)\,|\,g(\upsilon_1|V_z) = x_2\right) \quad \textit{with}\ g(\upsilon_1|V_z) = \left(\left(\left(\frac{V_z}{\gamma}\right)^{-\frac{\rho}{\upsilon}} - w_2\upsilon_1^{-\rho}\right)/w_1\right)^{-\frac{1}{\rho}}$$

where $g(\upsilon_1|V_z)$ is the isopoverty contour with regard to abscissa values x_1 (here income).

[9] Stata minimum 2DGAP ado files for the minimum and slope solution are available from the authors by request.

Relative 2DGAP The minimum 2DGAP c might be defined relative to the maximum 2DGAP distance $c_{\max}$, which is the distance from the origin (0,0) to the respective orthogonal slope of the IMD weak focus isopoverty threshold:

$$c_{rel} = c/c_{\max} \text{ where } c_{\max} = \|c_{\max}\| = \left[(p_1)^2 + (f(p_1, V_z))^2\right]^{0.5} = \min! \qquad (13)$$

The solution of $c_{\max}$ in Eq. 13 follows that of c of Eq. 9.

3.3 Minimum multidimensional poverty gap (minimum 2DGAP) – single poverty attribute intensities, aggregation and extensions

The *single poverty attribute intensities*, the distances a and b, are of particular interest since they disentangle and make transparent the single poverty attributes (say a for income measured in EURO, and b for time measured in minutes) while respecting their interdependence. They illustrate the respective compensation between the poverty attributes since their relation $\tan\alpha = b/a$ is the orthogonal slope (c') to the isopoverty line slope ($c'_\perp$, marginal rate of substitution) where the straight line c is crossing.

Once the distance c (Eq. 12) is found (solved via p_1) then with the actual $x = (x_1, x_2)$ and $p = (p_1, p_2 = f(p_1 | V_z))$ the single attribute gap intensities are easily calculated by

$$a = p_1 - x_1 \text{ and } b = f(p_1) - x_2 \qquad (14)$$

with its corresponding *relative single poverty attribute gap intensities*

$$a_{rel} = (a/a_{\max}) \text{ and } b_{rel} = (b/b_{\max}). \qquad (15)$$

where $a_{\max}$ and $b_{\max}$ are the single maximum poverty attribute intensities corresponding to $c_{\max}$ (Eq. 13).

Aggregation and mean minimum poverty 2DGAP The individual absolute or relative minimum 2DGAPs and their single multidimensional attributes might be aggregated and characterized by their means

$$C = \frac{1}{n_{IMD\ poor}} \sum_{i \in IMD\ poor} c_i,\ A = \frac{1}{n_{IMD\ poor}} \sum_{i \in IMD\ poor} a_i,\ B = \frac{1}{n_{IMD\ poor}} \sum_{i \in IMD\ poor} b_i, \qquad (16)$$

$$C_{rel} = \frac{1}{n_{IMD\ poor}} \sum_{i \in IMD\ poor} c_{rel,i},\ A_{rel} = \frac{1}{n_{IMD\ poor}} \sum_{i \in IMD\ poor} a_{rel,i},\ B_{rel} = \frac{1}{n_{IMDpoor}} \sum_{i \in IMDpoor} b_{rel,i}, \qquad (17)$$

where $c_{rel} = c/c_{\max}$, $a_{rel} = a/a_{\max}$ and $b_{rel} = b/b_{\max}$ are the respective relative 2DGAP intensities and their relative single multidimensional attributes according to Eq. 13.

The aggregation of the single poverty attributes a and b and of the 2DGAP c over all individuals might not result in the joint aggregate condition $C = (A^2 + B^2)^{0.5}$. Thus, there is no similar decomposability of the aggregated distance C according to Eq. 9 because without any additional information just a mean length of c has no unique end at the isopoverty threshold. However, with two degrees of freedom one remaining component (a, b or c) is computable from the other aggregates. In our application, alternative computations of the respective remaining component have shown close accordance to the orthogonal condition.

Another measure in the spirit of a Foster-Greer-Thorbecke type index is

$$C_\alpha = \frac{1}{n}\sum_{i=1}^{n}\left(max\left(\frac{c_i}{c_{\max}},0\right)\right)^\alpha, A_\alpha = \frac{1}{n}\sum_{i=1}^{n}\left(max\left(\frac{a_i}{a_{\max}},0\right)\right)^\alpha, B_\alpha = \frac{1}{n}\sum_{i=1}^{n}\left(max\left(\frac{b_i}{b_{\max}},0\right)\right)^\alpha \quad (18)$$

with n now as the total population number, $\alpha = 0$ for the multidimensional headcount ratio, $\alpha = 1$ as an average relative poverty gap but applied to the total population and $\alpha > 1$ respecting a higher aversion against multidimensional strong deprivations.

With respect to C_α this would be still in the spirit of a single well-being index of the multidimensional situation. However, with respect to A_α and B_α the single poverty dimensions will be transparent and the aggregated 2DGAP then provides a comprehensive multidimensional poverty picture with its disentangled components.

Minimum 2DGAP and the union approach In the union approach compensation is only defined for the intersection area (see Fig. 1) where the above discussed minimum poverty 2DGAP approach then is applicable. Any poverty situation beyond the single poverty thresholds, however, is reduced in the union approach to the respective poverty threshold via min(.) within the FGT multidimensional poverty index of Eq. 8. The multidimensional poverty gap there collaps to a unidimensional gap directly measurable as the unique distance between (z_1, x_2) and (z_1, z_2) respectively (x_1, z_2) and (z_1, z_2).

Other than a CES-type IMD weak focus poverty threshold If another than a CES-type IMD weak focus poverty threshold contour is preferred, then convexity of that function is required to apply the minimum 2DGAP (MDGAP) approach. Further possible requirements yet have to be developed.

Minimum MDGAP The minimum 2DGAP can be extended to the m -dimensional case, called minimum MDGAP, by a multivariate minimum search where the slopes of the MDGAP linear distance are subject to the orthogonality of the n dimensional tangents to the isopoverty threshold contour. A conceivable minimum 3DGAP for example would consider three dimensional isopoverty contours and a two dimensional tangent plane resulting in a minimum 3DGAP which is right-angled to the tangent plane.

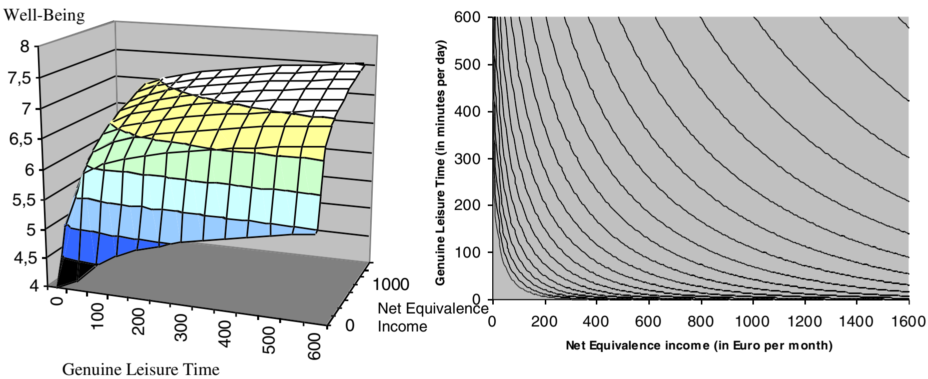

Fig. 3 Estimated CES well-being function – well-being and isopoverty contours for Germany 2001/02 Source: Own estimation with GSOEP 2002, active population

Minimum 2DGAP and dominance Consider the comparison of two poor persons in the two dimensional attributes space. Any person on the same multidimensional poverty isoquant is assigned to be poor at the same poverty well-being level. Persons with a respective isoquant farer away in the direction to the origin are assigned to be poorer. Depending on the narrowness according to different values of the poverty attributes of the isoquants (see Fig. 3) it is possible, that the 2DGAP (the distance c) of a less poorer person could be *longer* than the 2DGAP of a poorer person. However, if the multidimensional poverty isoquants would be parallel, then a longer 2DGAP would also describe a poorer person.

Yet, the virtue of our 2DGAP approach and our claim is not about dominance, but rather about transparency of the single attributes and their compensation under the interdependent well-being evaluation. So, for two different persons but with the same poverty well-being level the 2DGAP provides additional and different information about the relative importance and compensation of their single attributes (the relation between the distances a and b). In general, despite being at a same poverty well-being level different compensations depending on different attribute levels are made transparent by the 2DGAP approach, which would be blurred and invisible by the well-being indicator alone. This additional information and transparency then might allow different and targeted antipoverty policies according to the different poverty attributes, different poverty regimes, groups and attribute ranges.

4 Time and income interdependent multidimensional poverty – application concepts

The following application starts with the justification and definition of our time, income and CES well-being multidimensional threshold application concepts (Section 4). Section 5 then describes the data bases and presents all empirical results.

4.1 Time and income as multidimensional poverty attributes – application concept

Income as a poverty dimension Income as the dominant resource to acquire goods and services traditionally is the central dimension in applied poverty and well-being analyses. Since household members generally share their (net) income with each other, the unit of observation is commonly the household. To compare various households of different structures, sizes and needs an equivalized household income is used, which equates the household (net) income divided by the sum of equivalence weights to all household members. Internationally, the OECD established a scale which assigns – as we do – a weight of one to the household head, a weight of 0.5 to additional household members aged 15 years or older and a weight of 0.3 to all others.

Conventional income-based poverty analyses from the European Union judge a person as income poor if net equivalized household income is below 60 % of the respective median income of all households ([14], XV). This concept is adopted in the present study.

Time as a poverty dimension Time is a fundamental prerequisite enabling and restricting daily living activities. The quantity of available time is important for individual well-being simply by allowing or prohibiting desired activities. We therefore argue that time is of no less importance than income as a prominent well-being dimension and should be considered in multidimensional poverty analyses.

Though there are convincing reasons to consider time as a well-being and poverty dimension, time is rarely recognized as a poverty dimension. Yet a few scholars such as Burchardt

[15], Harvey and Mukhopadhyay [28] or Vickery [49] highlight the importance of considering non-market time in poverty analyses. Goodin, Rice, Parpo and Eriksson [26] determine "necessary time" and assume that the remainder – hidden in each single activity – is discretionary time, a concept which is hard to apply empirically. Other scholars like Zacharias [52], Calvo [16] or Bardasi and Wodon [5] arbitrarily define time poverty as a certain fraction of leisure respectively as a multiple of working hours as a poverty line but trade-offs between poverty attributes are beyond their focus.

We embrace the social participation and social inclusion/exclusion aspect by expanding the income poverty dimension with a specific time poverty dimension.[10] In particular, together with further economic and social perspectives, we argue that time (beyond income) is an elementary poverty dimension since social participation requires time. This corresponds to Sen's [42–44] extended perspective on poverty, since time, similarly to commodity, is a basic condition needed to accomplish any functioning to achieve a capability set with its respective freedom of choice.

In contrast to different leisure time poverty concepts as in the literature above we define *genuine personal leisure time* as the remaining available time left after all responsibilities/obligations as paid labour time, household working time, childcare, household requirements, sleeping, personal care and health activities are carried out. Time poverty then occurs when remaining genuine personal leisure time is below a certain level, and no or only restricted time is left for social participation.

The question arises whether time poverty should be considered in the household context as with income poverty. Since genuine personal leisure time can only be reallocated between household members to a very limited extent, it is most strictly linked to the individual. Thus, we regard time poverty as being at the personal rather than at a household/family level.

As a definition of a time poverty threshold, Bittman [8], p. 14 suggests a median concept similar to the traditional income orientated poverty concepts. Adapting this concept to EU standards, we use the 60 % median time poverty line.

4.2 Interdependence of multidimensional time and income poverty – CES well- being function application concept

Why should we care about for the interdependence of time and income? Time availability restricts all activities and requires activity allocation within the day, the week and other time periods. Market time to achieve income competes in any time period with other non-market activities including genuine personal leisure time. The more time is spent for income gaining purposes, the less is available for leisure and vice versa. This is the well known microeconomic trade-off of maximizing well-being (utility) with consumption and leisure as arguments subject to the time and income constraints yielding the optimal allocation of time for consumption and leisure. The shape of the well-being function in turn defines the degree of the substitution/compensation, i.e., the trade-off between labour and leisure time.[11]

Though the individual time/income optimal allocation is not in the focus of our analysis but the microeconomic approach illustrates the general competing time/income interdependence. How the compensation/substitution between income (consumption) and

[10] Economic implications of social cohesion are discussed e.g., in Osberg [38].

[11] Both time and income are crucial to the extended Becker [6] household production model as well. There a household maximizes the utility of final commodities which are produced in the household by utlizing market goods and time.

genuine personal leisure is quantified depends on the well-being function and its quantified parameters, which in turn is the task for our empirically based CES well-being estimates.[12]

Compensation/substitution of time and income is specified above by a CES well-being function (Eq. 3). As mentioned, in contrast to others (like [11] or [32]) who arbitrarily chose the dimension weights in their empirical applications, our CES well-being function parameters will be estimated using a population base. The idea and reasoning behind is to "let the people" evaluate the compensation between that time and income rather than any expert. Of course, who, and with which convincing argument, should designate the poverty line is an ongoing discussion within the social evaluation debate in general.

The estimation needs a well-being indicator as the dependent variable for estimation. A suitable indicator is individual satisfaction, for which data is available from the German Socio-Economic Panel (see next section) and used in our CES parameter estimation.

An individual then is assigned to be *interdependent multidimensional poor* (IMD poor, compensation approach, weak focus axiom) if his or her poverty attributes are below that IMD isopoverty threshold, which is the CES well-being isoquante crossing the intersection (z_1,z_2) of the unidimensional time and income poverty thresholds (see Fig. 1). Note, this is regardless of any voluntary or non-voluntarily individual well-being situation. As in common poverty analyses, an individual is counted to be poor if (s)he is below a defined poverty line.

5 Time and income multidimensional poverty – empirical results for Germany

With the application concepts at hand we now present time and income multidimensional poverty results for Germany. The data bases are briefly described followed by the empirical time, income and their multidimensional poverty lines for Germany. Then uni- and well-being multidimensional FGT and minimum 2DGAP results are presented and discussed.

5.1 Data: GSOEP and GTUS 2001/02

5.1.1 The German Socio-Economic Panel (GSOEP)

The German Socio-Economic Panel (GSOEP) provides representative individual longitudinal data for all persons older than 16 years living in German households. The representative panel study started in 1984 and provides subjective as well objective information about the individual living conditions in Germany [51]. In particular, the SOEP is asking for satisfaction with regard to different topics, like income as well as general question about life satisfaction. The 11-point scale general satisfaction information is used for our well-being estimation.

Since appropriate well-being data are only available within the German Socioeconomic Panel we use the GSOEP for the CES well-being estimation. Although in principle we could use the SOEP for our further analyses we prefer to use time use diary data from the actual available German Time Use Survey (GTUS) 2001/02 (with no appropriate well-being information) since the time use diaries provide more additional in-depth information. In

[12] For a further discussion of time and income see also Bonke, Deding and Lausten [9], Hamermesh and Pfann [27] with discussions on the economics of time use, and Merz [34] on time use and economic well-being.

particular only the diaries provide information about the time spent with others i.e. social participation.

5.1.2 The German Time Use Surveys (GTUS) 2001/02

In the actual available *German Time Use Survey (GTUS)* 2001/02 of the German Federal Statistical Office [22] all household members older than 11 years noted their daily routines in diaries using their own words for three days during the week, including two working days and a Saturday or Sunday. In addition to the subsequent coded information of the diaries personal and household supplements inform about the respective information. The GTUS 2001/02 sample for our analysis finally provides information of 5,144 households with 11,908 persons and 35,685 diaries.

Income is measured in the GTUS 2001/02 as monthly household net equivalence income, as described above. Time is measured as personal genuine leisure time. This time information, extracted in detail from the individual time use diaries, includes activities that are allocated to one of the main categories “Social Life and Entertainment”, “Participation in Athletic Activities e.g. Outdoor Activities”, “Hobbies and Games” and “Mass Media”.

5.2 Time, income and multidimensional empirical poverty lines

The active as well as the non-active population are the basis for the empirical determination of the single time and poverty thresholds and ensure comparability with other poverty analyses.[13] Likewise, it is debatable that certain population groups like the unemployed face different time situations or time evaluations. However, not to include them is arbitrary and neglects the modus operandi in common poverty analyses. In the following we present results for 2001/02. Results for 1991/92 and the discussion about poverty dynamics over a decade in Germany are available in the discussion paper Merz and Rathjen [36].

5.2.1 Time and income empirical poverty lines

Based on the above discussed applied concepts the empirical income, time and interdependent multidimensional poverty lines are given in Table 1.

The single *income poverty line* for Germany was €793.55 for 2001/02. The single *genuine personal leisure time poverty* line is 186 min for 2001/02.

A time poverty line of about 3 hours a day might seen by some to be relatively high. But this figure is the consequence of the evaluation by the active as well as the non-active population. It may be argued that the scope and kind of leisure is different between these groups, like that of the retired or unemployed compared to the active population, with the consequence of some exaggeration of the genuine leisure time poverty threshold.[14] However and discussed, for the *evaluation purpose* it would be arbitrary to exclude some groups for a society’s poverty threshold evaluation. Obviously, it is a normative decision to respect the society as a whole as the poverty threshold decision maker. Other approaches are conceivable and used in the income poverty discussion. Yet, consequences of those different decisions have to be analyzed in another study.

[13] If only the active population would be evaluated, income poverty would increase considerably

[14] The genuine personal leisure poverty threshold within our frame only for the active population would be 126 minutes per day and thus about one hour less than for the total population.

Table 1 Income, time and interdependent multidimensional empirical poverty lines, Germany 2001/02

	2001/02
Median net equivalence income (in € per month and 2002 prices)	1322.58
Median personal leisure time (in minutes per day)	310
Income poverty line (= 60 % median net equivalence income)	793.55
Time poverty line (= 60 % median personal leisure time)	186
$V^{poor} = f(I^{poor}, L^{poor})$	6.827

Own calculations with GTUS 2001/02, weighted data
Note: time and income poverty lines by GTUS data are calculated for the total (active and non-active) population

5.2.2 CES well-being estimates and IMD time and income empirical poverty line

Our evaluation of individual well-being with satisfaction data refers to the recent happiness/satisfaction literature [20, 24]. Such interpersonal utility comparisons are critically discussed particularly in economics. Yet, Van Praag and Ferrer-i-Carbonell [48] provide some convincing arguments for measuring well-being by instead using survey questions about satisfaction, as we do.

Using the German Socio-Economic Panel (SOEP) reported general life satisfaction 11-point scale[15] then for estimating individual well-being requires a type of ordered response modelling. However, as is well-known from production function estimation, the Kmenta [30] Taylor series approach allows a simple classical linear regression OLS estimator of the log transformed non-linear CES well-being function of Eq. 3 which is described in Merz and Rathjen [35]. Significant estimated coefficients together with the fulfilment of further consistency rules quantify the relevance of the substitution/compensation between time and income.

Specifying our CES relationship of Eq. 3 with personal genuine leisure time L and net equivalence income I as inputs and reported general life satisfaction as a proxy for the multidimensional well-being output V_i, the estimated CES well-being function for Germany 2002 is

$$V_i^{2002} = 3.550\left(0.519 I_i^{0.297} + 0.481 L_i^{0.297}\right)^{\frac{0.108}{0.297}} \tag{19}$$

with its contours as isopoverty lines as in Fig. 3.[16]

The population based evaluated substitution/compensation between genuine time and income results in a substitution elasticity of $\sigma = 1.422$. Thus, substitution is a bit “easier” than in the Cobb-Douglas type ($\sigma = 1$) situation.[17]

[15] SOEP 2002 question 11 in the personal questionnaire.

[16] The CES well-being function estimates take the working population into account, with the argument that the active population in particular actually experiences work and leisure and therefore judges the trade-off between the two dimensions more appropriate.

[17] Within a CES function the degree of substitution is measured by the substitution elasticity which ranges from perfect substitution (one unit of income substitutes one unit of time, $\rho = -1, \sigma = \infty$), over a certain degree of substitution (including the Cobb-Douglas case with ($\rho = 0, \sigma = 1$), to no substitution at all (either time or income poverty, no intersection, complementary input factors, $\rho \to \infty, \sigma \to 0$). For a further discussion of CES characteristics originated from production theory see Arrow et al. [3].

The evaluated *IMD poverty line* (weak focus) at the intersection of the unidimensional time and income thresholds is about a well-being level of 6.827 in 2001/02 based on the estimated parameter as in Eq. 9

$$V_{2002}^{poor} = f(I^{poor}, L^{poor}) = 3.550 \cdot (0.519 \cdot 793.55^{0.297} + 0.481 \cdot 186^{0.297})^{-\frac{0.108}{-0.297}} = 6.827. \quad (20)$$

The estimated input coefficients, the weight w for income and $(1 - w)$ for personal leisure (Eq. 3), indicate some dominance for income. However, the evaluated time contribution is not far away from a balanced 50/50 situation and refers to the importance of the time dimension as evaluated by the German population.

Given the estimated CES parameters we are also able to characterize the "correlation increasing switch" situation discussed in Section 2.1: the estimated CES well-being parameters fulfils the *non-decreasing poverty under correlation increasing switch* axiom for the multidimensional FGT measures with $\alpha = 0, \alpha = 1$ and $\alpha = 2$ as well as the further poverty axioms presented above.

5.3 Time, income and multidimensional poverty – empirical results for Germany

The empirical German poverty literature so far focuses on unidimensional[18] income inequality and income poverty (e.g., [7, 25]). Thus the following extends the empirical findings and provides new results for the multidimensional poverty case in Germany.

Whereas in our empirical definitions the respective poverty thresholds rely on the active and non-active population, the following poverty intensity analyses focus on the *active population*.[19] With the focus on the active population, our poverty analyses accentuate the situation of the working poor, a population group of growing interest and importance in discussions about the German labour market.[20]

Given the empirical thresholds, each GTUS sample person belongs to one of six multidimensional poverty regimes in Fig. 4. Figure 4 illustrates the IMD poverty risk as headcount ratios (Foster-Greer-Thorbecke (FGT) measure with $\alpha = 0$) in different poverty regimes. Table 2 provides the overall multidimensional FGT results including their uni- and multidimenional well-being poverty gaps, their respective standard errors, and 95 % bootstrapping confidence intervals.[21]

Unidimensional income poverty The percentage of income poor active individuals (headcount ratio / FGT index with $\alpha = 0$) is about 4.8 % (regimes 1, 2, 4, see Fig. 4). The corresponding poverty intensity index – measuring the average relative poverty gap – is about 1.07 %. Note, small gap figures reflect the FGT type division by the total population number (and not only by the number of poor people).

[18] Unidimensional poverty considers poverty only with one poverty dimension without any multidimensional concept.

[19] With more than 5 daily working hours (similar to the SOEP 2002 estimation) we avoid part-time situations with less restricted total leisure time.

[20] According to the household income concept, the working poor refer to an entire household and not necessary to a working poor person himself.

[21] The FGT with $\alpha = 1$ provides relative mean poverty gaps. Further absolute mean interdependent multidimensional and unidimensional time and income poverty gaps are given in Merz and Rathjen [36].

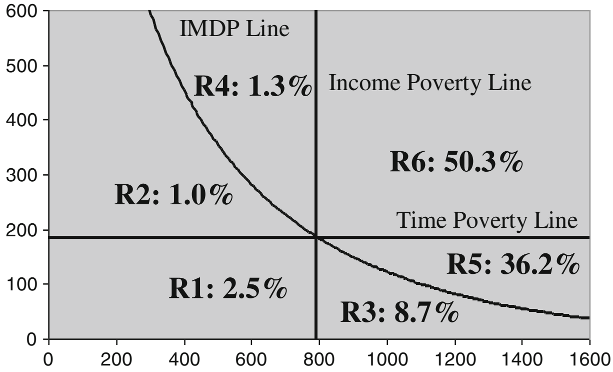

Fig. 4 Interdependent multidimensional and unidimensional poverty risk as headcount ratios in different poverty regimes, Germany 2001/02
Source: Own calculations with the GTUS 2001/02, weighted data, active population

Unidimensional time poverty (active population) The percentage of time poor active individuals (headcount ratio / FGT index ($\alpha = 0$)) is 47.4 % (see Table 2). For this group the time poverty level is remarkably high. The reason is that though we focus on the active population, the non-active population with its relative high leisure time yields a relatively high time poverty threshold and consequently a relative high time poverty ratio.[22]

The unidimensional time poverty gap measured by the FGT index with $\alpha = 1$ is 18.52 % which in relative terms is remarkably larger than in the income case.

Multidimensional poverty (union approach, strong focus axiom) According to the *union approach (strong focus axiom),* which defines poverty under the income respectively time poverty line (shaded area in Fig. 1), the percentage of multidimensional poor individuals is 49.7 % (regimes 1–5). Note that the relatively high level depends on incorporating all regimes under both thresholds, in particular inclusive regime 5, a compensation regime under weak focus. The gap intensity index of FGT with $\alpha = 1$ (FGT1) is 0.0125 (1.25 %).

Multidimensional poverty (compensation approach, weak focus axiom) According to the *compensation approach (weak focus axiom),* which allows a substitution between poverty dimensions also above the single dimension thresholds, the headcount ratio of the multidimensional poor individuals is 12.2 % (regimes 1, 2 3), which is a remarkable proportion of the active population. The FGT1 with 0.38 % is a considerably smaller well-being gap than witin the union approach and emphasizes again the importance of the assigned time and income compensation.

Poverty Regimes Figure 4 illustrates the varying importance of poverty regimes according the headcount ratios. Interestingly, the prominent portion of the IMD working poor is found in regime R3 with 8.7 %. There the time deficit is assigned not be compensated for even by above poverty income, and as such, is a distinct measure of the importance of time as a poverty dimension. With regime R1 we have a core IMD poverty of persons being time as well as income poor of 2.5 %.

[22] A similar argument holds for the relatively low income poverty ratio: The non-active population with no labour income diminishes the income poverty threshold which in turn yields a lower income poverty ratio.

Table 2 Interdependent multidimensional and unidimensional time and income poverty, Germany 2001/02

		Index	Std. Err.	95 % Conf. Interval	
FGT[1] ($\alpha = 0$)	Unidimensional				
	Income	0.04816	0.00342	0.04145	0.05487
	Time	0.47357	0.00721	0.45943	0.48771
	Multidimensional				
	Union (SF)[2]	0.49702	0.00745	0.48241	0.51163
	Compensation (WF)[2]	0.12159	0.00459	0.11260	0.13058
FGT ($\alpha = 1$)	Unidimensional				
	Income	0.01067	0.00092	0.00885	0.01248
	Time	0.18522	0.00371	0.17795	0.19248
	Multidimensional				
	Union (SF)	0.01254	0.00032	0.01191	0.01317
	Compensation (WF)	0.00378	0.00021	0.00336	0.00419
FGT ($\alpha = 2$)	Unidimensional				
	Income	0.00352	0.00038	0.00277	0.00427
	Time	0.10434	0.00273	0.09898	0.10970
	Multidimensional				
	Union (SF)	0.00073	0.00004	0.00065	0.00081
	Compensation (WF)	0.00027	0.00002	0.00022	0.00032

FGT = Forster-Greer-Thorbecke measure; [2] SF = strong focus axiom,WF = weak focus axiom, confidence intervals by bootstrapping
Source: Own calculations with GTUS 2001/02, weighted data, active population

To summarize the overall picture Unidimensional income and time poverty pinpoint the prominent time poverty risk and poverty intensity as a result of relatively high headcount ratios of almost 5 % (income poverty) respective almost 50 % (time poverty). This results in a calculation of the mean relative poverty gaps measured by FGT1 with 1.1 % (income poverty) and 18.5 % (time poverty). *Multidimensional poverty* under the strong focus axiom (union approach) is 49.7 % and reflects a much higher multidimensional poverty risk than that observed under the weak focus axiom (compensation approach) with 12.2 % (headcount ratio). The multidimensional poverty gap FGT1 and FGT2 measures are based on well-being units. Both multidimensional poverty gap measures illustrate significantly smaller intensities in the compensation approach compared to the union approach which emphasizes the importance of the assigned time and income compensation by the German Society.

5.4 Minimum multidimensional poverty 2DGAP

The multidimensional poverty gap above was measured using well-being units. Our proposed minimum multidimensional poverty 2DGAP, however, provides additional information about the single time and income attributes and disentangles the interdependent poverty dimensions respecting the evaluated substitution/compensation. As discussed, the minimum 2DGAP thus points to an optimal way out of multidimensional poverty by providing information about its single attributes. And, the 2DGAP components, the distances

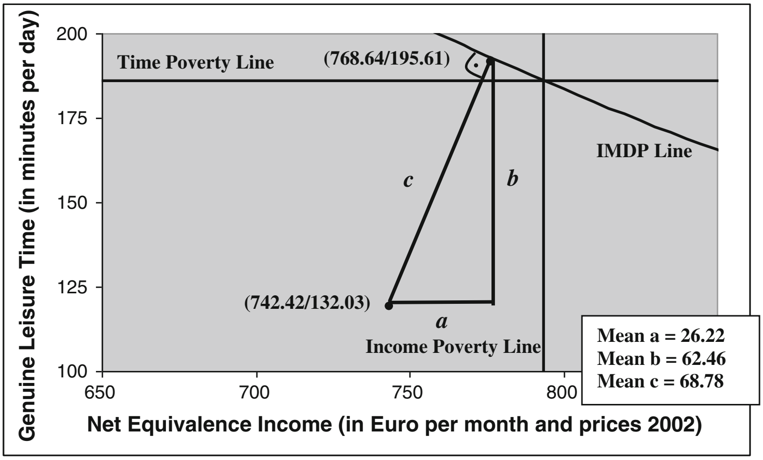

Fig. 5 Multidimensional poverty center of mean minimum multidimensional poverty 2DGAP, Germany 2001/02
Source: Own figure based on GTUS 2001/02, weighted data, active population

a and *b*, illustrate the compensation of the poverty attributes. This is important for targeted antipoverty policies.

Mean minimum multidimensional poverty 2DGAP The mean shortest way out of multidimensional poverty for Germany (distance *c* in Fig. 5 and Table 3) with respect to time and income in 2001/02 is about 68.78 units. The mean multidimensional poverty gap is relatively near the IMD poverty line: it is about 11.42 % of the maximum 2DGAP (which starts at point $(x_1, x_2) = (0, 0)$).

Figure 5 illustrates the mean situation of multidimensional poverty in Germany for 2001/02.

The mean minimum 2DGAP might characterizes a *center of IMD poverty* (compensation approach, weak focus axiom). It is defined by the coordinates of the starting point of the overall mean of all individual minimum 2DGAP distances *c* and its attributes *a* and *b*. The center coordinates then are found by iteration with the mean distances *a* and *c*, say, together with the orthogonality condition (as of Eq. 2).[23]

The mean minimum IMD poverty situation, the starting point of the distance *c*, is characterized by €742.42 and 132.03 min (see Fig. 5). The mean minimum 2DGAP meets the IMD poverty threshold at €768.64 and 195.61 min of genuine personal leisure time. Thus, on average about €26 of income, and a bit more than one additional hour of genuine personal leisure time is needed to escape multidimensional poverty (compensation approach, weak focus axiom) for Germany 2001/02.

Interestingly, the mean single poverty 2DGAP attributes of the time poverty gap (14.96 %, distance *b*) is comparably higher than the income poverty gap (6.04 %, distance *a*) emphasizing the peculiar importance of the time poverty component within multidimensional time and income poverty. In addition, the relation *b/a* of the 2DGAP poverty

[23]Though the respective empirical means of *a*, *b* and *c* do not necessarily exactly satisfy the Pythagorean theorem at the mean values, the results are close in our case when respective two of them are used in the calculation.

intensity components illustrates the compensation of the minimum 2DGAP center: with $c' = \tan(b/a) = 2.38$ (slope of c) and $c'_{\perp} = -1/c' = -a/b = -0.42$ (slope of the IMD isopoverty line where c is crossing, marginal rate of substitution). Thus the assigned amount of time to exchange one EURO income locally is about 0.42 min; i.e. it is less than a one to one compensation, 0.42 min are enough to compensate €1 which highlights the particular strength of the time burden.

Poverty regimes The mean minimum 2DGAP as an overall description of the multidimensional poverty situation has a starting point (IMD poverty center) in the core poverty regime (R1). This highlights the particular importance of the poverty intensity for the working poor as including time *as well as* income poverty (intersection, Fig. 5). This is the IMD poverty regime where the specific mean minimum 2DGAP with 152.57 is the highest. To escape poverty by a shortest path in 2001/02, one would need €72 and 133 min of genuine personal leisure time (Table 3).

The next important case is the R2 situation, where income deficits are assigned not to be compensated by genuine personal leisure time. To escape poverty for that 1 % of the working poor, one would on average need €47 and 58 min to emerge from IMD poverty. The smallest mean minimum poverty 2DGAP is that of regime 3. The regime R3 poor, with above poverty income while still being time poor, which is the largest IMD working poor group, would need on average €11 and 43 min to escape IMD poverty. Thus, concerning the mean minimum poverty 2DGAP the poverty intensity is quite different according to different regimes of compensation.

The particular importance of the time burden will also be visible when the median instead the respective mean is considered. The *median minimum 2DGAP center* is given by €997.15 income and 72.51 min and is no more in the intersection regime R1 but in the R3 regime, where the time deficit is assigned not to be compensated by above poverty threshold income.

IMD well-being vs. minimum 2DGAP poverty The multidimensional FGT poverty gap measures (FGT1 and FGT2) based on well-being units cover a variety of paths between different well-being levels and as a one value index the measures are no more transparent to its single poverty attributes. Compared to this, the unique path based on the 2DGAP approach between two well-being levels reveal the single poverty attributes and thereby add important

Table 3 Mean minimum poverty 2DGAP by poverty regimes, Germany 2001/02

	2DGAP: c	2DGAP: a	2DGAP: b
	Mean Minimum 2DGAP	Mean Minimum Income 2DGAP	Mean Minimum Time 2DGAP
IMD Poverty Regimes		(€)	(minutes per day)
R1	152.57 (5.19)[1]	72.18 (3.31)	133.50 (4.14)
R2	75.13 (7.17)	46.75 (5.01)	58.26 (5.19)
R3	44.34 (1.33)	10.82 (0.40)	42.88 (1.27)
IMD Poverty (WF)[2]	68.78 (2.00)	26.22 (1.12)	62.47 (1.69)
IMD Poverty (%)[3]	11.42	6.04	14.96

[1]Standard deviation in parentheses [2]WF = weak focus axiom [3]in % of the respective max 2DGAP: $c_{\max} = 602.51$, $a_{\max} = 434.40$, $b_{\max} = 417.51$,
Source: Own calculations with GTUS 2001/02, weighted data, active population

information to multidimensional poverty. Thus the mean minimum poverty 2DGAP and its respective single poverty intensity attributes with the above empirical results for Germany advocate for targeted antipoverty policies.

6 Further discussion and concluding remarks

This study analysed time and income interdependent multidimensional (IMD) poverty, and introduced a new multidimensional poverty gap, which we call the minimum poverty 2DGAP (MDGAP). This measure disentangles single attribute poverty intensities while respecting the compensation/trade-off of poverty attributes in a multidimensional approach. Whereas one value multidimensional well-being gap measures blur the multitude of gaps in the attribute space, the minimum 2DGAP explores the shortest gap as a unique path to escape poverty. Beyond the compact interdependent multidimensional poverty description by the mean minimum 2DGAP, there is the additional single dimension feature: each single attribute is transparent and measurable in the attribute's dimension, say income in EURO and time in minutes as in our application. This information and transparency then allows single dimension targeted antipoverty policies respecting its multidimensional interdependence.

The novel application of Germany 2001/02 of the active population (working poor) in our study was based on a CES well-being function estimated by satisfaction SOEP data. Thereby compensation between income and genuine personal leisure time was evaluated by the German population and not arbitrarily chosen. Genuine personal leisure time in particular takes up social participation as an indicator for social inclusion in an extended poverty perspective following Sen. Uni- and multidimensional poverty risk and poverty gap results (GTUS data) are presented with a well-being Foster-Greer-Thorbecke (FGT) and our new minimum poverty 2DGAP approach.

Without repeating the single results described above, it is important to note that the compensation approach respects the interdependence of both time and income poverty, and delivers significant and remarkable results for Germany which go beyond traditional understanding. Thus, although working more than 5 hours a day about 12 % of the working population is measured to be IMD poor applying the new approach. The percentage of the working that is poor in genuine personal leisure time *as well as* income is quite high (2.4 %). However, this is topped by those where time poverty is assigned not to be compensated with an income that is even higher than the income poverty threshold (8.7 %). This reflects the fact that the German society assigns a relatively high value to time and in particular to personal genuine leisure time including its social participation aspects.

Concerning the IMD poverty gap, the mean minimum poverty 2DGAP as a center of multidimensional time and income poverty is about 11.4 % of the maximum multidimensional poverty gap. Thus, the mean shortest way out of multidimensional poverty is characterized by single poverty 2DGAP attributes of 15.0 % for time, and 6.0 % for income emphasizing the peculiar importance of the time poverty component within multidimensional time and income poverty. The relation of the multidimensional 2DGAP attributes illustrates their compensation. With the respective marginal rate of substitution of about 0.42 min less than a one to one compensation is given at this poverty center which again highlights the particular strength of the time burden. Thus, targeted antipoverty policy should respect the different strengths of the multidimensional poverty attributes which is revealed by the population evaluation.

Since our analyses and results are novel for Germany, any comparison with the literature is not yet possible. This is the case for time poverty and the interdependent multidimensional time and income poverty, and also for the working poor, a group of particular economic and social policy interest.

Though a measure like the minimum 2DGAP indicates a short path out of poverty, real life conditions may restrict such a way to overcome poverty. And, with respect to efforts and costs other ways might be appropriate and different for different individuals. Nevertheless, the minimum poverty 2DGAP is proving to be a well-suited measure to distinctly characterize the multidimensional poverty situation with useful information for population anchored antipoverty policies.

All our empirically based results indicate the importance of the time dimension especially with its social particpation and ask to incorporate the time dimension into multidimensional poverty approaches. As for the German population evaluation, time is so valuable that a remarkable proportion of the working population are assigned not compensating for their time deficit, even by improving their income. Our approach with its social inclusion perspective is supported by further results not presented here which indicate that social participation of the time and income IMD poor indeed show less time in social participation activities than the IMD non-poor.

Any targeted policy for reducing poverty thus ignores an important dimension if time with its social exclusion aspects is neglected. Targeted antipoverty policies for the working poor beyond income policies (such as a minimum wage or other labour market policies), will require particular time policies e.g. for a better and more efficient synchronization of working and non-working time (concerning flexible working hours, commuting and public transport, childcare support, parental leave conditions, coordinated public services, etc.).

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