WORKING PAPER

Optimal grazing management rules in semi-arid rangelands with uncertain rainfall

by Martin F. Quaas and Stefan Baumgärtner

University of Lüneburg
Working Paper Series in Economics

No. 193

January 2011

www.leuphana.de/institute/ivwl/publikationen/working-papers.html

ISSN 1860 - 5508

Optimal grazing management rules in semi-arid rangelands with uncertain rainfall

MARTIN F. QUAAS[a*] and STEFAN BAUMGÄRTNER[b]

[a] Department of Economics, University of Kiel, Germany

[b] Department of Sustainability Sciences and Department of Economics, Leuphana University of Lüneburg, Germany

January 2011

Abstract. We study optimal adaptive grazing management under uncertain rainfall in a discrete-time model. As in each year actual rainfall can be observed during the short rainy season, and grazing management can be adapted accordingly for the growing season, the closed-loop solution of the stochastic optimal control problem does not only depend on the state variable, but also on the realization of the random rainfall. This distinguishes optimal grazing management from the optimal use of most other natural resources under uncertainty, where the closed-loop solution of the stochastic optimal control problem depends only on the state variables. Solving this unusual stochastic optimization problem allows us to critically contribute to a long-standing controversy over how to optimally manage semi-arid rangelands by simple rules of thumb.

JEL-Classification: Q57, D81, Q12

Keywords: environmental risk, risk management, stochastic optimal control, grazing management, rules of thumb

*Address: Department of Economics, University of Kiel, Wilhelm-Seelig-Platz 1, 24118 Kiel, Germany. Email: quaas@economics.uni-kiel.de.

1 Introduction

Semi-arid areas cover a large part of the Earth. They are characterized by low and highly variable precipitation. Livestock grazing is the predominant type of land use, providing the livelihood for more than a billion people. Yet, income from livestock grazing is associated with large uncertainties, as the productivity of the pastures depends strongly on the low and highly variable precipitation (Behnke et al. 1993, Sullivan and Rhode 2002, Westoby et al. 1989). In many semi-arid areas, in particular in Sub-Saharan Africa, the sparse yearly rainfall is concentrated in a distinct and short rainy season that is followed by the growing season of the grassland vegetation. For a risk-averse farmer, the challenge of rangeland management is to optimally adapt to this highly variable and highly uncertain rainfall, taking into account ecosystem dynamics.

More or less sophisticated grazing management strategies have been developed in South Africa and Namibia that adapt stocking rates to the amount of rainfall that has been observed in the current year's rainy season. They come in the form of rules that specify how to stock depending on the actual rainfall during the current year's rainy season. Specific rules that have been proposed by rangeland scientists include a low constant stocking density[1] (Lamprey 1983, Dean and MacDonald 1994), or "opportunistic" strategies that match the stocking rate with the available forage in every year (Behnke et al. 1993, Scoones 1994, Sandford 1994, Westoby et al. 1989). A more sophisticated strategy is to leave a fixed part of the pasture ungrazed in years with abundant rainfall ("resting in rainy years"), i.e. stocking is less than the grazing capacity of the pasture in such a year, while the pasture is used fully in years with low precipitation (Walter and Volk 1954, Rothauge 2007, Müller et al. 2007, Quaas

[1] In this paper, the *stocking density* refers to the number of livestock per hectare of rangeland while the *stocking rate* refers to the ratio of livestock number and available forage on the pasture in a given year.

et al. 2007, Baumgärtner and Quaas 2009). A number of studies have studied stylized, parametrized grazing management rules and attempted to determine optimal management rules by computing the parameter(s) of a given rule that maximizes some objective function (Janssen et al. 2004, Higgins et al. 2007, Hein and Weikard 2008). Both among scientists and farmers there is a long-standing controversy over which of these rules is more appropriate, and what difference rangeland or rainfall characteristics may make for that sake.

In this paper we study optimal adaptive grazing management under rainfall uncertainty in a simple rangeland model in discrete time. In contrast to most previous studies on grazing management in semi-arid areas, we fully determine the optimal grazing management rule by solving the stochastic dynamic optimization problem of a risk-averse farmer. A particular feature of rangeland management in semi-arid areas is the temporal structure within a year, with a distinct rainy season that provides the possibility to adapt the stocking rate to actual rainfall already in the current grazing season. Optimal management thus does not only depend on the current state of the vegetation, but also on the observed rainfall. In technical terms, the closed-loop solution of the stochastic optimal control problem does not only depend on the state variable, but also on the realization of the random variable. This distinguishes optimal grazing management from the optimal use of most other natural resources under uncertainty, for example fisheries or mineral ore extraction, where the closed-loop solution of the stochastic optimal control problem depends only on the state variables (e.g. Reed 1979, Pindyck 1984, Sethi et al. 2005, Costello and Polasky 2008). The other important difference to most studies on the use of stochastic natural resources is that we consider a risk-averse farmer. For risk-neutral decision makers, efficient management strategies have been derived before for fisheries (e.g. Reed 1979, Costello et al. 2001) and for semi-arid rangelands by Weikard and Hein (forthcoming), who show that the efficient stocking rate is not sustainable. Solving the stochastic optimization problem allows us to numerically derive the optimal grazing management rule for a risk-averse farmer, i.e. derive the optimal stocking rate as a function of the state of the rangeland and the current year's actual rainfall.

Thus, we can critically contribute to the long-standing controversy over how to optimally manage semi-arid rangelands by simple management rules. We show that an adaptive grazing management strategy with resting in years with good rainfall ("resting in rainy years") is optimal if the state of the vegetation is not extremely good and if rainfall is within reasonable limits. If the vegetation is in a very good condition, an "opportunistic" strategy with full stocking is optimal (Westoby et al. 1989). A low constant stocking density is not optimal, except for a very risk averse farmer and only in a range of intermediate rainfall and vegetation conditions. In any case, optimal grazing management strongly depends on the farmer's degree of risk aversion.

The paper is organized as follows. In the following Section 2, we describe the model. We analytically characterize optimal grazing management rules for the case of deterministic and constant rainfall in Section 3, and study optimal grazing management in the stochastic setting in Section 4. In Section 5, we discuss our results and conclude.

2 Stochastic model of semi-arid rangelands

We consider a model in discrete time that captures the periodic structure of the rangeland dynamics in many semi-arid areas, in particular in Sub-Saharan Africa. There is a distinct and relatively short rainy season that brings an uncertain amount of rainfall, followed by the growing season of the grassland vegetation. Given the state of the vegetation and the actual amount of rainfall during the rainy season, the farmer can estimate how much forage will be available on the rangeland for the year until the next rainy season. The time structure within a time period (a year) thus is the following: (i) rain falls, (ii) the farmer adapts the stocking rate, (iii) the state of vegetation develops depending on the previous state of vegetation, rainfall, and grazing pressure.

We consider two quantities to describe the grassland vegetation at each time t: green biomass G_t and reserve biomass R_t of a representative grass species.[2] The green biomass captures all photosynthetic ('green') parts of the plants, while the reserve biomass captures the non-photosynthetic reserve organs ('brown' parts) of the plants below or above ground (Noy-Meir 1982). The green biomass develops during the growing season in each year and dies almost completely in the course of the dry season. We therefore describe it as a flow variable on the yearly timescale. The quantity G_t of green biomass in year t after the end of the growing season depends on rainfall r_t in the current year, on the amount of reserve biomass R_t in that year, and on a growth parameter w_G. In particular, we assume that green biomass grows in proportion to rainfall, i.e. with a constant rain-use efficiency (Lehouerou 1984), and also in proportion to reserve biomass:

$$G_t = w_G \, r_t \, R_t \ . \tag{1}$$

Rainfall is stochastic and described by the independently and identically distributed (iid) random variable r_t. We assume a log-normal distribution of rainfall $r_t \sim LN(\mu_r, \sigma_r)$, as this is an adequate description for semi-arid areas (Sandford 1982). The log-normal distribution is determined by the mean μ_r and standard deviation σ_r of rainfall. We measure rainfall in units of ecologically effective rain events, i.e. the number of rain events during the rainy season with a sufficient amount of rainfall to be ecologically productive.

The reserve biomass R_t is a stock variable describing the state of the vegetation. It quantifies the parts of the grass that survive beyond the current year ('perennial grass'). Thereby, the dynamics of the vegetation is not only influenced by the current

[2]We assume that selective grazing is completely prevented, e.g. through rotational grazing with temporarily high grazing pressure, i.e. there is no competitive disadvantage for more palatable grasses (Beukes et al. 2002). Hence, we consider a single, representative species of grass.

precipitation, but also depends on the history of precipitation and grazing. The net growth of the reserve biomass from the current year t to the next one is described by the following equation of motion:

$$R_{t+1} - R_t = -d\, R_t \left(1 + \frac{R_t}{K}\right) + w_R\, G_t\, (1 - c\, x_t) \left(1 - \frac{R_t}{K}\right) . \qquad (2)$$

The first term on the right-hand side of this equation describes the natural mortality of grass. The parameter d is the intrinsic mortality rate. In addition, mortality is density dependent – the more reserve biomass, the faster it decreases due to natural mortality. The second term describes the natural growth of reserve biomass. Growth is driven by photosynthetic activity, which is assumed to be in proportion to green biomass. This process is also density dependent: the more reserve biomass, the slower is natural growth. The capacity limit K regulates both density dependencies.[3] The factor in the middle of this term on the right-hand side of (2) captures the impact of grazing.

We use x_t to denote the *stocking rate*, that is, the fraction of green biomass used as forage for livestock. Green biomass is used completely for grazing over the whole year if $x_t = 1$ and partly if $0 < x_t < 1$. Even with full stocking, $x_t = 1$, growth of reserve biomass is not necessarily reduced to zero. For, the parameter $c > 0$ that captures the impact of grazing on the growth of reserve biomass may be less than one. This is because green biomass is not eaten up immediately by the livestock, and thus has some time to contribute to reserve biomass growth.

The farmer's income is assumed to be proportional to the green biomass used as forage for livestock, $x_t G_t$. The underlying assumption is that both the value of marketed products (meat, milk, offspring) and the costs of farming accrue in proportion to the green biomass used as forage. This means in particular that we

[3]Note that K is not directly the "carrying capacity" of the rangeland (as e.g. in the logistic growth model), as the steady-state level of reserve biomass R even in the absence of grazing is below K. Yet, it constitutes an upper limit for reserve biomass R, as K would be the carrying capacity under rainfall conditions so extremely favorable that the effect of mortality can be neglected.

assume constant prices, which is an adequate assumption if products of livestock farming are sold at international markets where price uncertainty is uncorrelated to local rainfall. Using (1) and normalizing, income y_t is given by

$$y_t = x_t\, R_t\, r_t \ . \tag{3}$$

A *grazing management rule* $x(R_t, r_t)$ determines the stocking rate in year t as a function of the stock of reserve biomass and the actual amount of rainfall in the current year. The farmer chooses the grazing management rule $x(R_t, r_t)$ that optimizes the present value of utility,

$$\max_{x(R_t, r_r)} E\left[\sum_{t=1}^{\infty} \frac{y_t^{1-\eta}}{(1-\eta)\,(1+\delta)^{t-1}}\right] \quad \text{subject to (1), (2) and (3)} \ . \tag{4}$$

In the objective function, E denotes the expectancy operator over the path of rainfall events over the entire time horizon, and $\delta > 0$ denotes the farmer's discount rate. We assume a non-linear objective function, i.e. $\eta > 0$. This non-linearity takes account of the farmer's preferences for a more constant stream of income over time, as well as aversion against income risk. In the following, we will refer to the parameter η as the farmer's risk aversion.

The optimal solution of the stochastic control problem (4) is characterized by the Bellman equation

$$\begin{aligned} V(R_t) = E_t\Bigg[\max_{0\le x\le 1} \Bigg\{ & \frac{(r_t\, x\, R_t)^{1-\eta}}{1-\eta} \\ & + \frac{1}{1+\delta} V\left(R_t - d\, R_t \left(1 + \frac{R_t}{K}\right) + w\, R_t\, r_t\, (1 - c\, x) \left(1 - \frac{R_t}{K}\right)\right) \Bigg\}\Bigg] \ . \end{aligned} \tag{5}$$

Here, $V(R_t)$ denotes the value function, that is, $V(R_t)$ gives the maximal present value of utility that can be derived from the rangeland with state R_t of reserve biomass. In order to derive (5), we have used (1) in (2) and defined the new parameter $w = w_G\, w_R$. Furthermore, we have inserted the expression for income (3) in the utility function (4). The expectancy operator E_t in Equation (5) is taken over the distribution of rainfall in year t only.

We would like to emphasize that there is a significant difference between the stochastic optimization problem considered here and a typical stochastic programming problem: as rainfall is known in the year before the stocking rate is chosen, the expectancy operator E_t extends over the whole expression on the right-hand side of (5). The x is chosen such as to adapt to the observed rainfall event r_t.

3 Optimal grazing management rule with deterministic and constant rainfall

We start the analysis by considering the case of deterministic rainfall, i.e. zero standard deviation of rainfall, $\sigma_r = 0$. This enables us to derive some analytical insights into the steady-state properties of the model. In this section, we use r (without subscript) to denote the constant level of rainfall, i.e. $r \equiv r_t \equiv \mu_r$. In a steady state with a constant stocking rate $x^\star$, the reserve biomass reaches a steady-state level $R^\star$ that is found by using the condition $R_{t+1} = R_t = R^\star$ in (2):

$$R^\star = K \, \frac{w\, r\,(1 - c\, x^\star) - d}{w\, r\,(1 - c\, x^\star) + d} \, . \tag{6}$$

If natural growth of reserve biomass is very high compared to natural mortality, i.e. $w/d \gg 1$, the steady state reserve biomass approximately equals the capacity limit K. In any case, the steady-state stock of reserve biomass is smaller than the capacity limit, even in the case without grazing. It is easily checked that the steady-state reserve biomass is monotonically decreasing with the stocking rate $x^\star$.

With deterministic rainfall the optimal stocking rate can be determined by the standard open-loop control problem that is solved by applying the Hamilton formalism. Straightforward calculations (see Appendix A) show that the stock of reserve biomass $R^\star$ and the stocking rate $x^\star$ in an interior steady state are given by

$$R^\star = K \left[1 - \frac{\delta}{2\,[w\,r + d]} - \sqrt{\frac{2\,d}{w\,r + d} + \left[\frac{\delta}{2\,[w\,r + d]}\right]^2}\,\right] , \tag{7}$$

$$x^\star = \frac{1}{c} \left[1 + \frac{d}{w\,r} - \frac{2\,d}{w\,r} \left[\frac{\delta}{2\,[w\,r + d]} + \sqrt{\frac{2\,d}{w\,r + d} + \left[\frac{\delta}{2\,[w\,r + d]}\right]^2}\,\right]^{-1}\right] . \tag{8}$$

Note that neither the steady-state stock of reserve biomass nor the steady-state stocking rate depend on the risk aversion η.

As the stocking rate must not be negative or exceed one (in which case the available green biomass is entirely used as forage), corner solutions $x^\star = 0$ or $x^\star = 1$ are possible. The corner solution $x^\star = 1$ is obtained if the rainfall exceeds a level $\overline{r} > 0$, which is given in Appendix A (Equation A.15).

It is also possible that the reserve biomass reaches the lower boundaries $x^\star = 0$ or $R^\star = 0$. There are three candidates for rainfall levels at which one of the lower boundaries is reached. From Condition (8), we find that the optimal steady-state stocking rate in an interior solution would become zero at rainfall levels below $\underline{r} = (d - \delta)/w$ (see Appendix A). This is not a solution to the optimization problem, however, as the reserve biomass would be depleted at zero stocking rate already at levels of rainfall below a threshold $r_0 = d/w$ (cf. Condition 6). But even this threshold of rainfall does not apply, because a stocking rate $x_\star$ that would lead to depletion of the vegetation in the steady state is already optimal at rainfall levels below the threshold $r_\star = (d + \delta)/w$, which is found by setting $R^\star = 0$ in Condition (7). The corresponding optimal stocking rate is $x^\star = \delta/(c\,(\delta + d))$ (see Appendix A).

Result 1 *With deterministic and constant rainfall, an optimal steady state with positive reserve biomass exists if and only if rainfall exceeds the threshold level* $r_\star = (d + \delta)/w$. *The levels of the stocking rate* $x^\star$ *and reserve biomass* $R^\star$ *in such an interior steady state are given by* (8) *and* (7). *The optimal steady-state stocking rate* $x^\star$ *monotonically increases from 0 to 1 with the level of rainfall for rainfall levels*

$\underline{r} \leq r \leq \bar{r}$. *The optimal steady-state amount of reserve biomass* $R^\star$ *monotonically increases with the level of rainfall for* $r \geq r_\star$. *All these results are independent of the degree of risk aversion* η.

This result is illustrated in Figure 1, with parameter values as given in Table 1. These parameter values of the rangeland model (d, w, and c), together with a mean of $\mu_r = 1.2$ and standard deviation of $\sigma_r = 0.7$ ecologically effective rainfall events, are appropriate for rangelands in southern Namibia (Müller et al. 2007). The capacity limit K may be interpreted as the 'size' of the rangeland. We normalized K such that the steady-state stock of reserve biomass in the absence of grazing is one.

[Table 1 about here.]

[Figure 1 about here.]

As rainfall is assumed to be deterministic and constant, the closed-loop solution of the dynamic optimization problem (4) is a rule that determines the stocking rate as a function of the current stock of reserve biomass, $x(R_t)$. We determine this optimal management rule by numerically computing the value function as a solution of (5). For this sake, we employ the collocation method (Miranda and Fackler 2002), where the value function $V(R)$ is approximated by a finite linear combination of Chebychev polynomials.[4] Given the value function, the grazing management rule is determined by solving the ordinary optimization problem stated on the right-hand side of (5).

Figure 2 shows the resulting optimal grazing management rule for different values of risk aversion η. The constant level of rainfall r is set to $\mu_r = 1.2$, such that the steady-state reserve biomass without grazing would be $R^\star = 1$ (cf. Equation 6). The other parameter values are as given in Table 1.

[Figure 2 about here.]

[4] The optimization routines were implemented in Matlab. All program codes are available from the authors upon request.

As seen in the figure, the optimal stocking rate is an increasing function of reserve biomass. The steady-state level of reserve biomass is apparent from the figure as the point where the grazing management rules for different values of η intersect. This is due to the fact that the steady state is independent of risk aversion (Result 1). The effect of risk aversion η on optimal grazing is also apparent from the figure. The lower the degree of risk aversion η, the more steeply does the optimal stocking rate x increase with reserve biomass R. For relatively small values of risk aversion η, the corner solution $x(R) = 1$ is reached at levels of the reserve biomass below the steady-state value without stocking, $R^\star = 1$. For a high risk aversion, full stocking is not optimal even for a pristine pasture with $R = 1$. Rather, the optimal stocking rate in this case is below full stocking.

Result 2 *With deterministic and constant rainfall, the optimal stocking rate x is an increasing function of reserve biomass R. The lower the degree of risk aversion η, (i) the more steeply it increases with reserve biomass, (ii) the higher the optimal stocking rate for below-steady-state-levels of reserve biomass, (ii) the lower the optimal stocking rate for above-steady-state-levels of reserve biomass, and (iv) the lower the level of reserve biomass from which on full stocking is optimal.*

The intuition for this result is as follows. A steeper grazing management rule means that the steady state is approached more quickly, but also at higher intertemporal inequalities in resulting income. A fast increase in reserve biomass if it is below the steady state is due to the low stocking rate that, however, leads to a low income at the same time. On the other hand, a fast decrease in reserve biomass if it is above the steady state is due to the high stocking rate that leads to a high income at the same time. A farmer who does not care so much about intertemporal fluctuation, but who is impatient, prefers the faster, but intertemporally less equal path to the steady state.

4 Optimal grazing management rule with stochastic rainfall

With stochastic rainfall, the optimal grazing management rule depends not only on the state of reserve biomass, R_t, but also on the actual amount of rainfall in the current year, r_t. This dependency on the random variable's realization in the current year is what distinguishes optimal grazing management from the optimal use of most other natural resources under uncertainty (see Section 1).

We determine the optimal grazing management rule by numerically solving the Bellman equation (5). For this sake, we employ a modified version of Miranda and Fackler (2002)'s collocation method, implemented in Matlab, that allows the optimal grazing management rule to depend on the realization of the random rainfall event.

[Figure 3 about here.]

In Figure 3 the resulting optimal grazing management rule is shown for two different values of risk aversion $\eta \in \{0.6, 1.0\}$; the other parameter values are given in Table 1. Lighter shades of gray in the contour plots indicate a lower stocking rate.

For both values of risk aversion, the optimal stocking rule depends on both the stock of reserve biomass and the amount of current rainfall if (i) the stock of reserve biomass is not too high and (i) the amount of rainfall is relatively low. The question of particular interest is, how the optimal stocking rate depends on the level of rainfall. This is the question we focus on in the following analysis.

If the stock of reserve biomass is high and and the farmer is not very risk averse (left graph in Figure 3), full stocking is optimal irrespective of the amount of rainfall. This corresponds to an opportunistic grazing management strategy that matches the stocking rate with the available forage in every year, irrespective of the amount of rainfall (see Section 1). Thus, we have the following result.

Result 3 (opportunistic grazing) *For a farmer with a relatively low degree of* *risk aversion, and for high levels of reserve biomass, an opportunistic grazing management strategy is optimal, i.e. full stocking is optimal irrespective of the amount of rainfall.*

For a lower level of reserve biomass, or for a more risk-averse farmer, the optimal stocking changes with actual rainfall, provided the rainfall is above some threshold value. To see this effect more clearly we consider how the optimal stocking rate depends on rainfall, keeping the level of reserve biomass fixed. Figure 4 shows the optimal stocking rate as a function of current rainfall only, with the stock of reserve biomass fixed at $R^\star = 0.36$, which is the steady state reserve biomass under deterministic rainfall (cf. Figure 2). As evident from the figure, the optimal stocking rate depends on current rainfall in the following way. If rainfall is low, full stocking is optimal, i.e. $x(R^\star, r_t) = 1$ for r_t smaller than some threshold level $r_{\min}$. At a threshold level of about $r_{\min} = 0.76$, the optimal stocking rate starts to decrease steeply with an increasing amount of rainfall. This decrease is the steeper the higher the risk aversion is. As a consequence, for all levels of current rainfall above the threshold, the optimal stocking rate is the higher, the lower the risk aversion is. If risk aversion is very low, the optimal stocking rate reaches a minimum and increases again when current rainfall is very high. In contrast, for higher levels of risk aversion the optimal stocking rate monotonically decreases with the level of current rainfall. The general finding, which is qualitatively the same for the different degrees of risk aversion considered, is stated in the following result.

[Figure 4 about here.]

Result 4 (resting in rainy years) *The optimal grazing management rule implies full stocking if current rainfall is below some threshold r_{min}. For rainfall above this threshold, the optimal stocking rate is less than one, i.e. a part of the pasture is left ungrazed in years with abundant rainfall.*

The intuition behind this result is the following. A high level of actual rainfall implies a large amount of green biomass in the current year. With a high stocking rate, this translates into a very high income in the current year. With a lower stocking rate, the remaining green biomass enhances the growth of reserve biomass, which increases the expected income in the following year. For a risk-averse farmer it is optimal to make use of this buffering function of the reserve biomass by employing a grazing management strategy with resting in rainy years. In years with good rainfall the farmer does not fully exploit the grazing capacity of the farm and thereby he shifts income to the next year with possibly worse conditions. The more risk averse the farmer is, the higher is the benefit from this buffering function and the more resting he applies in rainy years. For a very low degree of risk-aversion, and a very high level of current rainfall, it is however optimal to use a larger part the green biomass immediately and let only part of it build up the stock of reserve biomass.

For a very low level of actual rainfall, this line of reasoning does not apply, however, because at a low level of current income, a risk-averse values current income very high compared to expected future income. Thus, full stocking is optimal below a certain threshold of actual rainfall. For a level of reserve biomass of $R^\star = 0.36$, as shown in Figure 4, this threshold level is $r_{\min} = 0.76$. The comparison with Figure 3 shows that this threshold level $r_{\min}$ of current rainfall below which full stocking rate is optimal depends on the stock of reserve biomass: The higher the current level of reserve biomass, the higher is $r_{\min}$.

When the farmer's degree of risk aversion is not very low, the optimal stocking *rate*, which is the ratio of livestock number and available forage on the pasture in a given year, decreases with the level of current rainfall. At the same time, given the stock of reserve biomass, the available forage on the pasture increases with current rainfall (cf. Equation 1). Thus, it may be hypothesized that the optimal stocking *density*, which is the number of livestock per hectare of rangeland, is (more or less) constant, i.e. independent of rainfall. Such a constant stocking density has been proposed as the adequate grazing management strategy in the literature (see Section 1). To explore the validity of this hypothesis, we consider the optimal stocking density as a function of actual rainfall in Figure 5, again for a stock of reserve biomass of $R^\star = 0.36$. As evident from the figure, the optimal stocking density is in general *not* independent of the level of actual rainfall, but rather an increasing and convex function of actual rainfall. An exemption may be the case of an extremely risk-averse farmer ($\eta \gg 1.5$), and for actual rainfall above the threshold $r_{\min}$. This finding is stated in the following result.

Result 5 (constant stocking density) *Optimal stocking density varies with both reserve biomass and rainfall. Only in the limiting case of extremely high risk aversion, stocking density approximately independent of rainfall, provided rainfall is above some threshold level.*

[Figure 5 about here.]

5 Conclusion and discussion

In this paper, we have analyzed a stochastic and dynamic model of rangeland management in semi-arid areas, and determined the optimal grazing management rule for a risk-averse farmer. Due to the particular intra-annual temporal structure of rangeland grazing, with a distinct rainy season that provides the possibility to adapt the stocking rate to actual rainfall already in the current grazing season, this optimal grazing management rule depends on both the current state of the vegetation, measured by the stock of reserve biomass in the model considered here, and on the level of actual rainfall in the current year.

Our results critically contribute to a long-standing controversy over how to optimally manage semi-arid rangelands by simple rules of thumb. We have shown that for high stocks of reserve biomass, and for farmers with a relatively low degree of risk aversion, an "opportunistic" strategy is optimal that matches the stocking rate with the available forage in every year. Thus, our analysis supports the recommendation of Behnke et al. (1993), Scoones (1994), Sandford (1994), Westoby et al. (1989) for farms where the rangeland is in a very good condition.

For lower stocks of reserve biomass, however, the optimal grazing management strategy is to apply a lower stocking rate in years in which current rainfall exceeds some threshold. In years with current rainfall below this threshold, on the other hand, full stocking is optimal. This result supports the "resting in rainy years"-grazing management strategies that are applied by some farmers in southern Africa, especially under marginal agricultural conditions (Walter and Volk 1954, Rothauge 2007, Müller et al. 2007, Quaas et al. 2007, Baumgärtner and Quaas 2009). The intuition for this result is that such adaptive grazing management rules with resting in years with abundant rainfall have a buffering function: they smooth income from livestock farming over time. Thus, the more risk-averse the farmer is, the more he will prefer a grazing management strategy with resting in rainy years. For a farmer with a very low degree of risk aversion, on the other hand, who does not care as much for this buffering function, the optimal strategy is to choose a higher stocking rate again in years with a very large amount of rainfall.

Our analysis suggests that a low constant stocking density is not a particularly good recommendation for grazing management. Except, possibly, for an extremely risk-averse farmer a constant stocking density is never optimal.

Moreover, our results for the case of deterministic and constant rainfall suggest that ongoing climate change may have important consequences for the sustainability of rangeland farming. If mean rainfall decreases due to climate change, it may become optimal to choose a stocking rate that degrades the grassland in the long run.

References

Baumgärtner, S., Quaas, M. F., 2009. Ecological-economic viability as a criterion of strong sustainability under uncertainty. Ecological Economics 68, 2008–2020.

Behnke, R. H., Scoones, I., Kerven, C., 1993. Range ecology at disequilibrium: New models of natural variability and pastoral adaption in African savannas. Overseas Development Institute, London.

Beukes, P. C., Cowling, R. M., Higgins, S., 2002. An ecological economic simulation model of a non-selective grazing system in the Nama Karoo, South Africa. Ecological Economics 42, 221–242.

Costello, C., Polasky, S., 2008. Optimal harvesting of stochastic spatial resources. Journal of Environmental Economics and Management 56 (1), 1–18.

Costello, C., Polasky, S., Solow, A., 2001. Renewable resource management with environmental prediction. Canadian Journal of Economics / Revue canadienne d'Economique 34 (1), 196–211.

Dean, W., MacDonald, I., 1994. Historical changes in stocking rates of domestic livestock as a measure of semi-arid and arid rangeland degradation in the Cape Province, South Africa. Journal of Arid Environments 26, 281–298.

Hein, L., Weikard, H.-P., 2008. Optimal long-term stocking rates for livestock grazing in a Sahelian rangeland. African Journal of Agricultural and Resource Economics 2 (2), 126–150.

Higgins, S., Kantelhardt, J., Scheiter, S., Boerner, J., 2007. Sustainable management of extensively managed savanna rangelands. Ecological Economics 62 (1), 102–114.

Janssen, M. A., Anderies, J. M., Walker, B. H., 2004. Robust strategies for managing rangelands with multiple stable attractors. Journal of Environmental Economics and Management 47, 140–162.

Lamprey, H., 1983. Pastoralism yesterday and today: The overgrazing problem. In: Bourlière, F. (Ed.), Tropical Savannas: Ecosystems of the World. Elsevier, Amsterdam, pp. 643–666.

Lehouerou, H., 1984. Rain use efficiency - a unifying concept in arid-land ecology. Journal of Arid Environments 7 (3), 213–247.

Miranda, M., Fackler, P., 2002. Applied Computational Economics and Finance. MIT Press, Cambridge, MA.

Müller, B., Frank, K., Wissel, C., 2007. Relevance of rest periods in non-equilibrium rangeland systems: A modelling analysis. Agricultural Systems 92, 295–317.

Noy-Meir, I., 1982. Stability of plant-herbivore models and possible applications to savanna. In: Huntley, B., Walker, B. (Eds.), Ecology and tropical savannas. Springer, Berlin, pp. 591–609.

Pindyck, R. S., 1984. Uncertainty in the theory of renewable resource markets. Review of Economic Studies LI, 289–303.

Quaas, M. F., Baumgärtner, S., Becker, C., Frank, K., Müller, B., 2007. Uncertainty and sustainability in the management of rangelands. Ecological Economics 62, 213–234.

Reed, W. J., 1979. Optimal escapement levels in stochastic and deterministic harvesting models. Journal of Environmental Economics and Management 6, 350–363.

Rothauge, A., 2007. Some principles of sustainable rangeland management in Namibia. Agricola 17, 7 – 15.

Sandford, S., 1982. Pastoral strategies and desertification: Opportunism and conservation in dry lands. In: Spooner, B., Mann, H. (Eds.), Desertification and Development. Academic Press, London, pp. 61–80.

Sandford, S., 1994. Improving the efficiency of opportunism: New directions for pastoral development. In: Scoones, I. (Ed.), Living with Uncertainty: New Directions in Pastoral Development in Africa. Intermediate Technology Publications, London, pp. 174–182.

Scoones, I., 1994. New directions for pastoral development in Africa. In: Scoones, I. (Ed.), Living with Uncertainty: New Directions in Pastoral Development in Africa. Intermediate Technology Publications, London, pp. 1–36.

Sethi, G., Costello, C., Fisher, A., Hanemann, M., Karp, L., 2005. Fishery management under multiple uncertainty. Journal of Environmental Economics and Management 50 (2), 300–318.

Sullivan, S., Rhode, R., 2002. On non-equilibrium in arid and semi-arid grazing systems. Journal of Biogeography 29, 1595–1618.

Walter, H., Volk, O. H., 1954. Grundlagen der Weidewirtschaft in Südwestafrika. Eugen Ulmer, Stuttgart.

Weikard, H.-P., Hein, L., forthcoming. Efficient versus sustainable livestock grazing in the Sahel. Journal of Agricultural Economics, doi: 10.1111/j.1477–9552.2010.00275.x.

Westoby, M., Walker, B. H., Noy-Meir, I., 1989. Opportunistic management for rangelands not at equilibrium. Journal of Range Management 42, 266–274.

A Appendix: Optimal grazing management in the deterministic case

The current-value Hamiltonian of the deterministic open-loop control problem is:

$$H = \frac{[r\,x_t\,R_t]^{1-\eta}}{1-\eta} + \lambda\left[-d\,R_t\left[1+\frac{R_t}{K}\right] + w\,R_t\,r_t\,[1-c\,x_t]\left[1-\frac{R_t}{K}\right]\right]. \qquad \text{(A.1)}$$

For an interior solution, the first-order conditions are

$$\frac{[r\,x_t\,R_t]^{1-\eta}}{x_t} = \lambda_t\,w\,r\,R_t\,c\left[1-\frac{R_t}{K}\right] \tag{A.2}$$

and

$$\frac{[r\,x_t\,R_t]^{1-\eta}}{R_t} + \lambda_t\left[-d\left[1+2\,\frac{R_t}{K}\right] + w\,r\,[1-c\,x_t]\left[1-2\,\frac{R_t}{K}\right]\right]$$
$$= \delta\,\lambda_t - [\lambda_{t+1}-\lambda_t]\ . \tag{A.3}$$

In a steady state we have $\lambda_{t+1} - \lambda_t = 0$. Omitting the time subscripts, and using (A.2) in (A.3), we obtain

$$w\,r\,c\,x\,\frac{R}{K} - d\left[1+2\,\frac{R}{K}\right] + w\,r\left[1-2\,\frac{R}{K}\right] = \delta\ . \tag{A.4}$$

From the condition for the steady-state reserve biomass R, Equation (6), we have

$$x = \frac{1}{c}\left[1-\frac{d}{w\,r}\,\frac{1+\frac{R}{K}}{1-\frac{R}{K}}\right]\ . \tag{A.5}$$

Using this result in (A.4), we get

$$[w\,r+d]\left[1-\frac{R}{K}\right]^2 - \delta\left[1-\frac{R}{K}\right] - 2\,d = 0\ . \tag{A.6}$$

The solution of this quadratic equation is

$$1-\frac{R}{K} = \frac{\delta}{2\,[w\,r+d]} \pm \sqrt{\frac{2\,d}{w\,r+d} + \left[\frac{\delta}{2\,[w\,r+d]}\right]^2} \tag{A.7}$$

$$R = K\left[1-\frac{\delta}{2\,[w\,r+d]} \mp \sqrt{\frac{2\,d}{w\,r+d} + \left[\frac{\delta}{2\,[w\,r+d]}\right]^2}\,\right]\ . \tag{A.8}$$

Since R cannot be greater than K, we obtain the result (7). Using (7) in (A.5), we find the steady-state stocking rate as given in (8). From Equation (7), we see that $R^\star$ is zero if rainfall is below a level $r_\star$ given by

$$\left[1-\frac{\delta}{2\,[w\,r_\star+d]}\right]^2 = \frac{2\,d}{w\,r_\star+d} + \left[\frac{\delta}{2\,[w\,r_\star+d]}\right]^2\ , \tag{A.9}$$

$$r_\star = \frac{\delta+d}{w}\ . \tag{A.10}$$

From (A.5), we conclude that this is the case at a stocking rate

$$x_\star = \frac{1}{c}\left[1 - \frac{d}{w\,r_\star}\right] = \frac{\delta}{c\,(\delta + d)} \tag{A.11}$$

From (8), we conclude that the corner solution $x = 0$ would be obtained at rainfall levels below a value of $\underline{r}$, which is determined by

$$\left[1 + \frac{d}{w\,\underline{r}}\right]\left[\frac{\delta}{2\,[w\,\underline{r} + d]} + \sqrt{\frac{2\,d}{w\,\underline{r} + d} + \left[\frac{\delta}{2\,[w\,\underline{r} + d]}\right]^2}\,\right] = \frac{2\,d}{w\,\underline{r}}\ . \tag{A.12}$$

The solution to this equation is

$$\underline{r} = \frac{d - \delta}{w}\ . \tag{A.13}$$

The corner solution $x = 1$ is obtained at rainfall levels above $\overline{r}$, which is determined by

$$\left[1 - c + \frac{d}{w\,\overline{r}}\right]\left[\frac{\delta}{2\,[w\,\overline{r} + d]} + \sqrt{\frac{2\,d}{w\,\overline{r} + d} + \left[\frac{\delta}{2\,[w\,\overline{r} + d]}\right]^2}\,\right] = \frac{2\,d}{w\,\overline{r}}\ . \tag{A.14}$$

This condition is a quadratic equation in $\overline{r}$ and has the unique positive solution

$$\overline{r} = \frac{2\,c\,d - (1-c)\,\delta + \sqrt{(4 - 8\,(1-c)\,c)\,d^2 - 4\,(1-c)\,d\,\delta + (1-c)^2\,\delta^2}}{2\,(1-c)^2\,w}\ . \tag{A.15}$$

For the corner solution $x = 1$, we have the condition for steady-state reserve biomass

$$d\left[1 + \frac{R}{K}\right] = w\,r\,(1-c)\left[1 - \frac{R}{K}\right]\ , \tag{A.16}$$

$$R = K\,\frac{w\,r\,(1-c) - d}{w\,r\,(1-c) + d}\ . \tag{A.17}$$

Without grazing, the reserve biomass would become zero at a rainfall level below r_0, which is given by the condition

$$R = K\,\frac{w\,r_0 - d}{w\,r_0 + d} = 0\ , \tag{A.18}$$

i.e. $r_0 = d/w$.

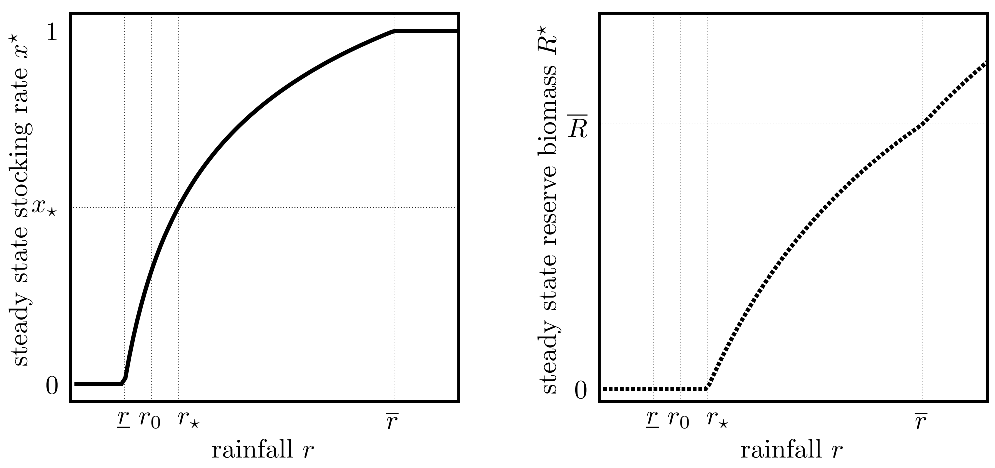

Figure 1: Optimal steady-state stocking rate and reserve biomass for different constant levels of rainfall. For the parameter values given in Table 1 the threshold levels of rainfall are $\underline{r} = 0.417$, $r_0 = 0.625$, $r^\star = 0.833$, and $\overline{r} = 2.50$.

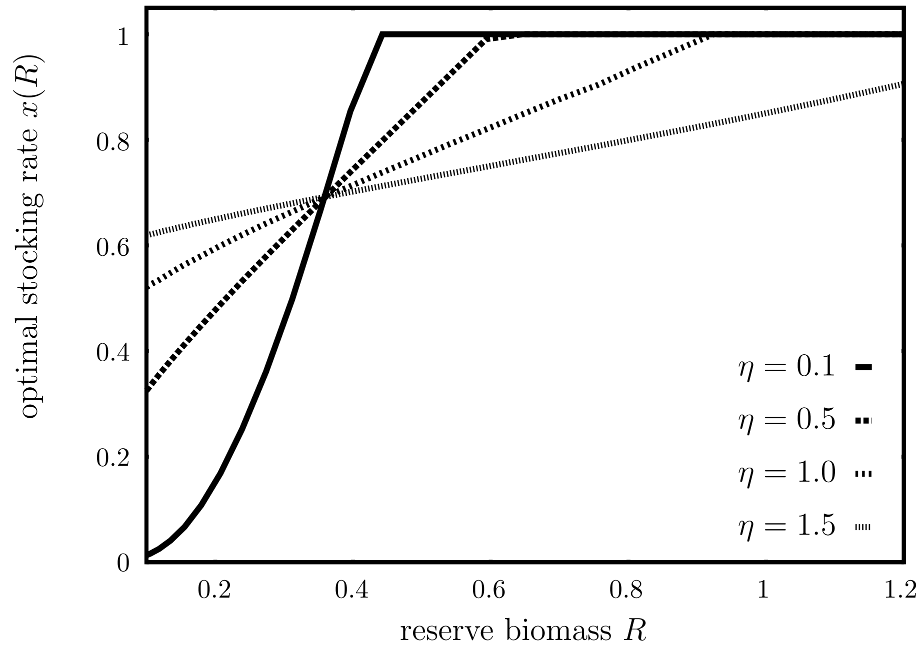

Figure 2: The optimal stocking rate under constant rainfall $r = 1.2$ for different values of risk aversion η. The other parameter values are given in Table 1.

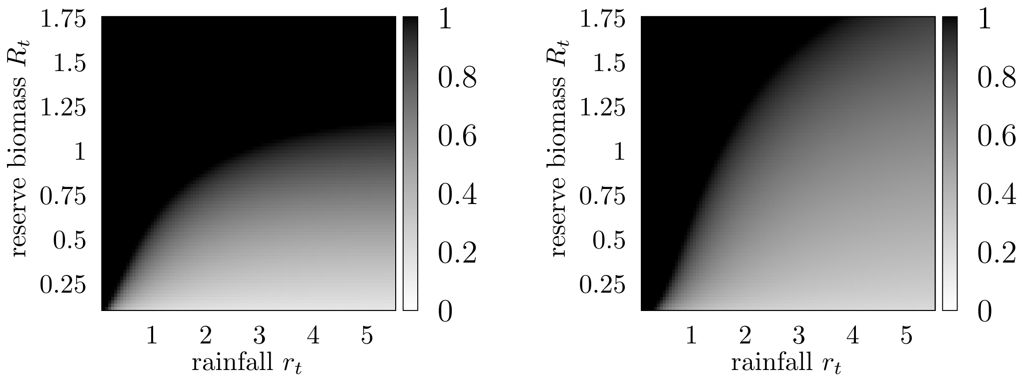

Figure 3: The optimal stocking rate (grayscale given in the panels right besides the graphs) under uncertainty for different values of risk aversion: $\eta = 0.6$ in the left graph and $\eta = 1.0$ in the right graph. The other parameter values are given in Table 1.

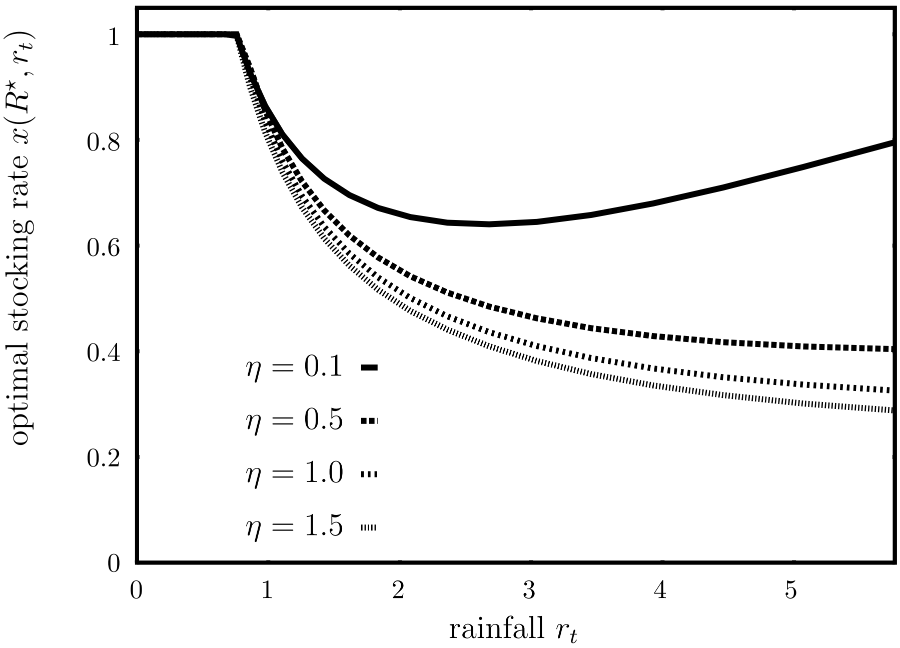

Figure 4: The optimal stocking rate x as a function of current rainfall r_t under uncertainty for different values of risk aversion η for a given stock of reserve biomass $R^\star = 0.36$. Parameter values are given in Table 1.

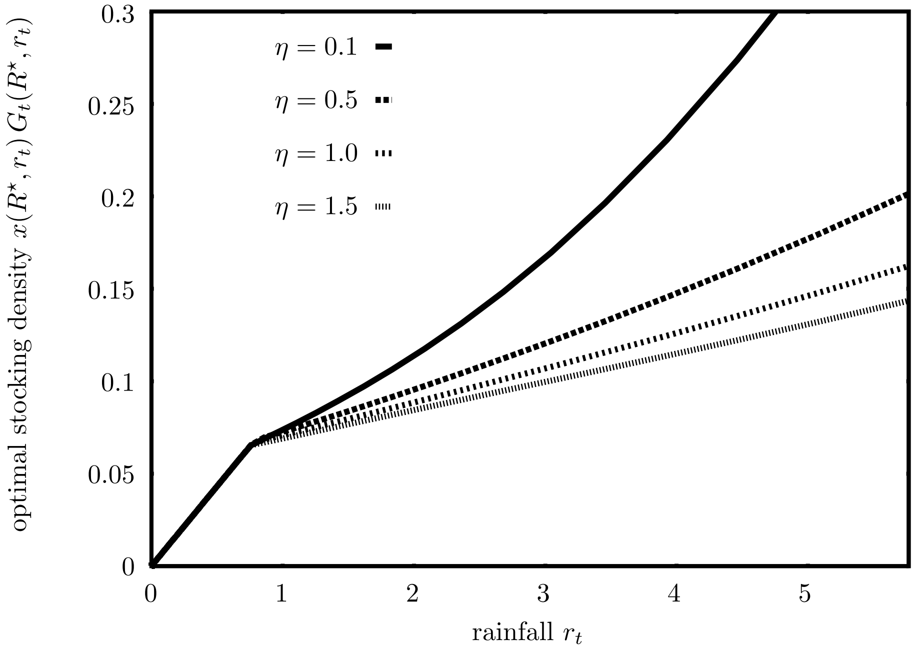

Figure 5: The optimal stocking density $x(R^\star, r_t)\,G_t(r_t)$ as a function of current rainfall r_t under uncertainty for different values of risk aversion η for a given stock of reserve biomass $R^\star = 0.36$. Parameter values are given in Table 1.

Parameter	Symbol	Value
mean rainfall	μ_r	1.2
standard deviation of rainfall	σ_r	0.7
intrinsic mortality rate	d	0.15
intrinsic growth rate	w	0.24
grazing impact	c	0.5
capacity limit	K	3.17
discount rate	δ	0.05

Table 1: Parameter values used in the numerical computations.

Working Paper Series in Economics

(recent issues)

No.192: Institut für Volkswirtschaftslehre: Forschungsbericht 2010, Januar 2011

No.191: *Natalia Lukomska, Martin F. Quaas and Stefan Baumgärtner*: Bush encroachment control and risk management in semi-arid rangelands, December 2010

No.190: *Nils Braakmann:* The causal relationship between education, health and health related behaviour: Evidence from a natural experiment in England, November 2010

No.189: *Dirk Oberschachtsiek and Britta Ulrich:* The link between career risk aversion and unemployment duration: Evidence of non-linear and time-depending pattern, October 2010

No.188: *Joachim Wagner:* Exports and Firm Characteristics in German Manufacturing industries, October 2010

No.187: *Joachim Wagner:* The post-entry performance of cohorts of export starters in German manufacturing industries, September 2010

No.186: *Joachim Wagner:* From estimation results to stylized facts: Twelve recommendations for empirical research in international activities of heterogenous firms, September 2010

No.185: *Franziska Dittmer and Markus Groth:* Towards an agri-environment index for biodiversity conservation payment schemes, August 2010

No.184: *Markus Groth:* Die Relevanz von Ökobilanzen für die Umweltgesetzgebung am Beispiel der Verpackungsverordnung, August 2010

No.183: *Yama Temouri, Alexander Vogel and Joachim Wagner:* Self-Selection into Export Markets by Business Services Firms – Evidence from France, Germany and the United Kingdom, August 2010

No.182: *David Powell and Joachim Wagner:* The Exporter Productivity Premium along the Productivity Distribution: First Evidence from a Quantile Regression for Fixed Effects Panel Data Models, August 2010

No.181: *Lena Koller, Claus Schnabel und Joachim Wagner:* Beschäftigungswirkungen arbeits- und sozialrechtlicher Schwellenwerte , August 2010

No.180: *Matthias Schröter, Markus Groth und Stefan Baumgärtner:* Pigous Beitrag zur Nachhaltigkeitsökonomie, Juli 2010

No.179: *Norbert Olah, Thomas Huth and Dirk Löhr:* Monetary policy with an optimal interest structure, July 2010

No.178: *Sebastian A. Schütz:* Structured Finance Influence on Financial Market Stability – Evaluation of Current Regulatory Developments, June 2010

No.177: *Franziska Boneberg:* The Economic Consequences of One-third Co-determination in German Supervisory Boards: First Evidence from the German Service Sector from a New Source of Enterprise Data, June 2010
[forthcoming in: Schmollers Jahrbuch / Journal of Applied Social Science Studies]

No.176: *Nils Braakmann:* A note on the causal link between education and health – Evidence from the German short school years, June 2010

No.175: *Torben Zülsdorf, Ingrid Ott und Christian Papilloud:* Nanotechnologie in Deutschland – Eine Bestandsaufnahme aus Unternehmensperspektive, Juni 2010

No.174: *Nils Braakmann:* An empirical note on imitative obesity and a puzzling result, June 2010

No.173: *Anne-Kathrin Last and Heike Wetzel:* Baumol's Cost Disease, Efficiency, and Productivity in the Performing Arts: An Analysis of German Public Theaters, May 2010

No.172: *Vincenzo Verardi and Joachim Wagner:* Productivity premia for German manufacturing firms exporting to the Euro-area and beyond: First evidence from robust fixed effects estimations, May 2010

No.171: *Joachim Wagner:* Estimated capital stock values for German manufacturing enterprises covered by the cost structure surveys, May 2010
[published in: Schmollers Jahrbuch / Journal of Applied Social Science Studies 130 (2010), 3, 403-408]

No.170: *Christian Pfeifer, Simon Janssen, Philip Yang and Uschi Backes-Gellner:* Training Participation of an Aging Workforce in an Internal Labor Market, May 2010

No.169: *Stefan Baumgärtner and Martin Quaas:* Sustainability Economics – general versus specific, and conceptual versus practical, May 2010
[forthcoming in: Ecological Economics]

No.168: *Vincenzo Verardi and Joachim Wagner:* Robust Estimation of Linear Fixed Effects Panel Data Models with an Application to the Exporter Productivity Premium, April 2010

No.167: *Stephan Humpert:* Machen Kinder doch glücklich? April 2010

No.166: *Joachim Wagner:* Produktivität und Rentabilität in der niedersächsischen Industrie im Bundesvergleich. Eine Benchmarking-Studie auf der Basis vertraulicher Firmendaten aus Erhebungen der amtlichen Statistik, April 2010
[erschienen in: Statistische Monatshefte Niedersachsen, Sonderausgabe "Kooperation Wissenschaft und Statistik - 20 Jahre Nutzung von amtlichen Mikrodaten", S. 30 - 42]

No.165: *Nils Braakmann:* Neo-Nazism and discrimination against foreigners: A direct test of taste discrimination, March 2010

No.164: *Amelie Boje, Ingrid Ott and Silvia Stiller:* Metropolitan Cities under Transition: The Example of Hamburg/ Germany, February 2010

No.163: *Christian Pfeifer and Stefan Schneck:* Relative Wage Positions and Quit Behavior: New Evidence from Linked Employer-Employee-Data, February 2010

No.162: *Anja Klaubert:* "Striving for Savings" – religion and individual economic behavior, January 2010

No.161: *Nils Braakmann:* The consequences of own and spousal disability on labor market outcomes and objective well-being: Evidence from Germany, January 2010

No.160: *Norbert Olah, Thomas Huth und Dirk Löhr:* Geldpolitik mit optimaler Zinsstruktur, Januar 2010

No.159: *Markus Groth:* Zur Relevanz von Bestandseffekten und der Fundamentalen Transformation in wiederholten Biodiversitätsschutz-Ausschreibungen, Januar 2010

No.158: *Franziska Boneberg:* Die gegen das Drittelbeteiligungsgesetz verstoßende Aufsichtsratslücke existiert. Replik zu „Das Fehlen eines Aufsichtsrates muss nicht rechtswidrig sein" von Alexander Dilger, Januar 2010
[erschienen in: Zeitschrift für Industrielle Beziehungen, 1 (2010)]

No.157: *Institut für Volkswirtschaftslehre:* Forschungsbericht 2009, Januar 2010

No.156: *Alexander Vogel, Joachim Wagner, Kerstin Brunken und Arno Brandt:* Zur Beschäftigungsentwicklung in der Region Hannover - Ein Vergleich mit 12 deutschen Verdichtungsräumen, Dezember 2009

No.155: *Nils Braakmann and Joachim Wagner:* Labor market adjustments after a great import shock: Evidence from the German clothing industry and the Multi-Fibre Arrangement, December 2009

No.154: *Joachim Wagner:* Zehn Jahre *European Data Watch*: Dokumentation von Datensätzen für die empirische Wirtschafts- und Sozialforschung und Zugangswegen zu den Daten, Dezember 2009
[erschienen in: AStA - Wirtschafts- und Sozialstatistisches Archiv 4(2010), 2, 141-149]

No.153: *Joachim Wagner:* Offshoring and work performance: Self-Selection, effects on performance, or both? December 2009
[revised version forthcoming in: Review of Word Economics]

No.152: *Christian Pfeifer:* Effective Working Hours and Wages: The Case of Downward Adjustment via Paid Absenteeism, November 2009

No.151: *Christian Pfeifer:* Adjustment of Deferred Compensation Schemes, Fairness Concerns, and Hiring of Older Workers, November 2009

No.150: *Franziska Boneberg:* Recht und Realität von Mitbestimmung im westdeutschen Dienstleistungssektor: 11 Fallstudien, November 2009

No.149: *Birgit Müller, Martin Quaas, Karin Frank and Stefan Baumgärtner:* Pitfalls and potential of institutional change: Rain-index insurance and the sustainability of rangeland management, November 2009

No.148: *Alexander Vogel, Florian Burg, Stefan Dittrich und Joachim Wagner:* Zur Dynamik der Export- und Importbeteiligung deutscher Industrieunternehmen – Empirische Befunde aus dem Umsatzsteuerpanel 2001-2006, Oktober 2009
[publiziert in: Wirtschaft und Statistik, Heft 11(2009), 1109-1116]

No.147: *Markus Groth:* Potentiale und Risiken der Nutzung von Methan aus Methanhydraten als Energieträger, Oktober 2009

No.146: *Sandra Derissen, Martin Quaas and Stefan Baumgärtner:* The relationship between resilience and sustainable development of ecological-economic systems, October 2009
[forthcoming in: Ecological Economics]

No.145: *Anne-Kathrin Last und Heike Wetzel:* Effizienzmessverfahren – Eine Einführung, September 2009

No.144: *Horst Raff and Joachim Wagner:* Intra-Industry Adjustment to Import Competition: Theory and Application to the German Clothing Industry, September 2009
[published in: The World Economy 33 (2010), 8, 1006-1022]

No.143: *Nils Braakmann:* Are there social returns to both firm-level and regional human capital? – Evidence from German social security data. September 2009

No.142: *Nils Braakmann and Alexander Vogel:* How does economic integration influence employment and wages in border regions? The case of the EU-enlargement 2004 and Germany's eastern border, September 2009

No.141: *Stefanie Glotzbach and Stefan Baumgärtner:* The relationship between intra- and intergenerational ecological justice. Determinants of goal conflicts and synergies in sustainability policy. September 2009
[forthcoming in: Environmental Values]

No.140: *Alexander Vogel:* Exportprämien unternehmensnaher Dienstleister in Niedersachsen, September 2009

(see www.leuphana.de/institute/ivwl/publikationen/working-papers.html for a complete list)

Leuphana Universität Lüneburg
Institut für Volkswirtschaftslehre
Postfach 2440
D-21314 Lüneburg
Tel.: ++49 4131 677 2321
email: brodt@leuphana.de

www.leuphana.de/institute/ivwl/publikationen/working-papers.html